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INVESTIGATION OF CONSTRUCTION AND MAINTENANCE OF AIRDROMES ON ICE 1953 - 1954

SELECTED EXCERPTS FROM

ICE CROSSINGS

BY

G. R. BERGMAN and B. V. PROSKURIAKOV

Trudy Nauchno-Issledovatel'skikh Uchreshdenii,
Seriia IV, Gidrometeorizdat, Moscow, 1943

translated by

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for

ARCTIC CONSTRUCTION AND FROST EFFECTS LABORATORY
NEW ENGLAND DIVISION
BOSTON, MASSACHUSETTS
under
OFFICE OF THE CHIEF OF ENGINEERS
AIRFIELDS BRANCH
ENGINEERING DIVISION
MILITARY CONSTRUCTION

OCTOBER 1954

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FOREWORD TO TRANSLATION

The material presented herein was translated by the SIPRE Bibliography Project, Library of Congress, Washington, D. C., through arrangement with the Snow, Ice and Permafrost Research Establishment, Corps of Engineers, U. S. Army. The translation was the work of Mr. Nikolay T. Sikejew.

The translated material consists of selected portions of the complete book, "Ice Crossings", by G. R. Bregman and B. V. Proskuriakov.(1) This book predates the later Russian book, "Data on the Problem of Ice Crossings", edited by B. L. Lagutin(2) and translated by the Stefannson Library for the Arctic Construction and Frost Effects Laboratory in 1950. The latter book, while more complete and advanced than the earlier work, makes frequent reference to "Ice Crossings". The Arctic Construction and Frost Effects Laboratory selected those portions of the book to be translated which covered phases of the subject in greater detail or which were not covered at all in the later work.

The translation has been retained in fairly literal form. However, some editorial footnotes have been added to clarify places which the reader might find difficult to follow due to errors in printing or mathematical formulation, or the use of unorthodox engineering terminology, in the original Russian book.

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(1) Ice Crossings, G. R. Bregman and B. V. Proskuriakov, Nauchno-Issledovatel'skikh Uchrezhdenii, serria IV, vypusk5, Gidrometeoizdat, Moscow, 1943.

(2) Data on the Problem of Ice Crossings, edited by B. L. Lagutin, Trudy Nauchno-Issledovatel'skikh Uchrezhdenii, Seriia V, Vypusk 20, Glavneye Upravlenixe Gidrometerologicheskoy Sluzhyly Gidrometeoizdat Sverdlovsk, Moscow, 1946.

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Chapter I

PROBLEM OF DETERMINING SAFE LOADS ON AN ICE COVER ANI CONSTRUCTION OF ICE CROSSINGS

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The problem of the supporting power of an ice cover is not solved, regardless of the frequent use of an ice cover for crossing. Even safe loading is determined without adequate considerations. Data presented in table 1 are most useful for this purpose. There, the safe load is considered only as a function of ice thickness. This assumes an invariability of the mechanical properties of an ice cover with time and with changes in the hydrometeorological conditions.

However, Professor N. Bernshtein demonstrated even in 1929, from Hertz's investigation (deflection of a plate on an elastic foundation), that safe load values on ice depend on:

1) radius of load distribution.
2) bending strength of ice.
3) ice thickness.
4) modulus of ice elasticity.
5) Poisson's ratio.

Professor Bernshtein calculated the data on the basis of the theory of elasticity alone. If we also take into consideration the elastic properties of ice, then a number of factors determining the supporting power of an ice cover must be considered further. Plastic properties and viscosity of ice provide for a change in the modulus of elasticity with time. The modulus, according to investigations of some authors, also depends on amount of load and ice temperature.

The relationships have not been studied enough, but the first attempts at their determination have been made. Hess, for an ice temperature range between 0° and -6.8°C, suggested the following equations:

For Moderate Loads

$$\frac{d\alpha}{dt} = \frac{c}{t} \quad (1)$$

$$\alpha = a + bt^2 \quad (2)$$

$$\frac{d\alpha}{dt} = 2bt \quad (3)$$

Table 1.

The Minimum Safe Ice Thickness of Ice-Crossings

Type of load	Total Weight in tons	Pressure on axle in tons: rear	Pressure on axle in tons: front	Minimum calculated values of ice thickness in cm.	Minimum distance between loads in m.
1. Soldier	0.1	-	-	5	5.0
Infantry in column	-				
2. Singly	-			7	7.0
3. In two's	-			7	7.0
4. In four's	-			10	10.5
5. Individual on horseback	0.5			10	10.5
Cavalry in column					
6. Singly				12	12.0
7. In three's				15	15.0
8. Cart	0.8			15	15.0
9. Loads on wheels	3.5	2.7	0.75	15	15.0
10. " " "	6.0	4.0	2.0	20	20.0
11. " " "	10.0	7.0	3.0	25	25.0
12. " " "	15.0	10.0	5.0	30	30.0
13. Loads on caterpillar tracks	3.5			15	15.0
14. " " "	10.0			20	20.0
15. " " "	12.5			25	25.0
16. " " "	25.0			40	40.0
17. " " "	45.0			50	50.0

where: α = relative ice deformation.

t = time

c = value, depending on load, size of samples and on time.

a = elastic deformation.

b = coefficient, determined by load and size of sample.

Thus, the intensity of ice deformation with time decreases for small loads, and increases for heavy loads. Royen's investigations indicated that deformation with any load increased with time, but that the rate of deformation increase diminishes with time according to the following equation:

$$\alpha = \frac{c\sigma\sqrt[3]{h}}{1+\theta} \tag{4}$$

where: c = coefficient equal 60-90x 10^{-5}

σ = load (in kg./sq. cm.)

θ = temperature of sample.

It may be noted in discussing the suggested formulas that ice viscosity should be taken into consideration when calculating icecover deformation.

The second and also very important factor, is the dependence of the mechanical properties of ice, and in particular the temporary resistance of ice on the temperature. Some of these interrelations are suggested in table 2.

Table 2.

Values of temporary resistance of ice depending on its temperature

Ice temperature (θ°C)	Bending strength in kg./sq. cm. according to:			
	Korzhavin	Vitman	Arnol'd-Alîab'ev	Veĭnberg
0	70	80	93	100
- 5	148	150	140	124
-10	224	190	177	140
-20	-	-	238	160

The supporting power of an ice cover is a function of many variables, including: the thickness and fracturing of ice, meteorological conditions, wind and thermal influences, uneven distribution of snow cover, etc. Thus, data of table 1 showing the relation between load and ice thickness might only be correct in some range of variables for determining the strength of an ice cover. These data cannot be recommended for practical use in every case.

Inasmuch as the number of variables is great, it is more correct to consider some combinations of these variables and then the table will indicate the real values of ice-supporting power.

Besides the table of minimum ice thickness for various transportation of loads, some principles for calculating the supporting power of an ice cover were introduced by others (Korunov, Zubov, Shuleĭkin).

M. M. Korunov proposed that the equation for deflection of a plate along a cylindrical surface is analogous to the equation for a beam which, if correct, gives for the same bending stress the relation between to loads Q_1 and Q_2 and the required ice thickness (H).

$$\frac{H_2^2}{H_1^2} = \frac{Q_2}{Q_1} \tag{5}$$

Korunov, from experimental data of ice thickness (H), load (Q) and equation (5) suggested the following formula:

$$H = 10\sqrt{Q} \tag{6}$$

Thus, Korunov neglected the variations of the elasticity modulus, the temporary resistance, and the load distribution along the ice surface, which are also very important. He applied the equations of cylindrical deflection of a plate to cases of single load movement. These simplifications cannot be approved from a practical viewpoint. Consequently, the equation of Korunov is not useful for calculations. Professor N. N. Zubov took into account the value of permissible deflections of an ice cover (not bending stress), that is different in principle. He determined the form of deflection by a logarithmic equation, which is closer to observational data. Naturally, the first derivative of linear deflection is discontinuous at the point of load application. The second derivative and value of the

bending moment at the point of load application are indeterminate.

Professor V. V. Shuleĭkin, and similarly Bernshteĭn, suggested the applicability of the results of the elasticity theory, and the assumption that the modulus of elasticity and temporary resistance of ice have constant values for studying ice-supporting power. In the present study, these last two assumptions are omitted and ice characteristics are analyzed according to the influence of hydrometeorological factors.

Many experiments in laboratories and under natural conditions were carried out to study ice-cover strength. However, the results are inconsistent. The values of temporary resistance and the elasticity modulus suggested by various authors are different by as much as a factor of ten. These differences might be explained by the ice property permitting a change in its physico-mechanical characteristics with temperature and with sea ice, salinity and ice age. These conditions were seldom considered in experiments.

Measures for ice-cover reinforcement are insufficiently developed due to the low reliability of calculated data.

Chapter 2

BRIEF CHARACTERISTICS OF THE CONDITIONS ON BODIES OF WATER AND THE PHYSICO-MECHANICAL PROPERTIES OF ICE AND SNOW.

1. Brief characteristics of ice conditions.

Cooling of a water surface by cold air causes a decrease in the water temperature to the freezing point. If water is in a quiet state, as for example in lakes, the cooling occurs at the surface layers first. Water temperatures in rapidly flowing rivers reaches the freezing point almost simultaneously through the entire cross section due to turbulent motion. Processes of water cooling in slowly moving rivers are similar to conditions observed in lakes. Water motion due to wind in large lakes and seas occurs not to the bottom but to a depth of several meters.

These peculiarities in water bodies determine the nature of ice formation processes in running and quiet water.

Rivers. The first appearance of ice in rivers occurs near the banks where the depth is less, and consequently, less time is needed for water to cool to 0°C. Low speed of flow near the coast hampers vertical exchange of water masses, thus water particles cooled to 0°C remain near the water surface where ice crystallization begins. The presence of favorable conditions for surface ice formation near the coast causes the appearance of land-ice, the width of which is determined by cooling intensity and speed of flow.

Distribution of water at 0°C in the stream causes ice formation in the middle of rivers. It occurs most frequently at the surface. Intensive turbulent mixing and supercooling of the whole water mass results in ice formation within and near the bottom of streams.

Ice crystals freezing together produce the following different initial forms of ice:

(a) thin ice, frazil ice;
(b) snowflakes frozen together, snow sludge;
(c) floating anchor ice, sludge;*
(d) floating ice fields, ice drift.

---

* Anchor ice growing up to the surface on river rapids results in the appearance of ice islands "pyatry".

Ice fields floating downstream and freezing together become thicker. Intensity of ice drifting increases during further cooling. Ice fields are stopped in narrow river sections, diverting the river near islands and sandbanks. When the ice stoppage withstands further pressure from following ice fields and the flow, then freeze-up results. A polynya is formed downstream from the stoppage and upstream freeze-up begins.

If river sections from the frozen edge carry masses of ice below the surface, then in the region of ice stoppage, great ice jams are formed. Ice jams sometimes occur for many kilometers of the river and frequently cause flooding. Ice jam regions are characterized by formation of hummocky ice and piled coast ice, reaching several meters in height.

Growth of the ice cover after freeze-up occurs mainly from the lower part. The intensity of ice growth at the lower edge depends on the heat exchange between the ice surface and the air. The presence of heat exchange results in heat flow from the water to the lower side of the ice cover (Q_1) and from the lower side of the cover to the upper. The latter heat flow is equal to the value of heat exchange between the upper ice surface and the atmosphere (Q_2).

The value Q_1 depends entirely on the water temperature and hydraulic characteristics of the water body, speed of flow, degree of turbulent mixing, etc. The value Q_2 is determined chiefly by meteorological conditions.

The sum of the heat flows, indicated by Q_1 and Q_2, is related to the intensity of ice growth by the following relation:

$$Q_1 + Q_2 = \frac{dH}{d\tau} L \cdot S \tag{7}$$

where: H = ice thickness

τ = time

L = heat of ice formation

S = ice density

Analysis of the values Q_1 and Q_2 yields the following equations:

$$Q = \alpha t \tag{8}$$

where: α = coefficient depending on the characteristics of the lower surface of the ice cover, the speed of flow, and on the water temperature. However, because the water temperature during the winter varies little, it is possible to consider the coefficient as depending on the first two factors only;

t = temperature of water in ^{o}C

$$Q_2 = q_1 + q_2 + q_3, \tag{9}$$

where: q_1 = heat loss from the upper ice surface by convection,

q_2 = similarly by evaporation,

q_3 = similarly by radiation.

and also:

$$q_1 = A\,(\theta - \gamma) \tag{10}$$

where: A = coefficient of heat exchange depending on wind velocity;

θ = air temperature;

γ = temperature of the upper ice surface.

$$q_2 = B\,(f - f_{\gamma}) \tag{11}$$

where: B = coefficient of heat exchange, depending on wind velocity;

f = absolute humidity;

f_{γ} = vapor pressure at the ice-surface temperature.

$$q_3 = C\left[\left(\frac{T_\theta}{100}\right)^4 - \left(\frac{T_\gamma}{100}\right)^4\right] \tag{12}$$

where: T_θ = air temperature in degrees absolute;

T_γ = temperature of the upper ice surface in degrees absolute;

C = radiation coefficient

The well-known formula of Devik may be obtained from equation (7) using the relations in (8), (10), (11) and (12). The heat flow Q_2 must be taken for the snow cover surface in cases when the ice is covered by snow.

Many formulas have been suggested for calculating ice cover growth. They are based on theoretical considerations. However, empirical coefficients for these were determined for rivers where the writers made their

experiments. Thus, the formulas have limited application. They do not take into account the importance of snow in the processes of heat exchange, "air-ice".

The State Hydrological Institute proposed the following general function for ice cover thickness on the given factors:

$$H = \left(\Sigma t^{\circ}_{-}, \Delta t^{\circ}_{+}, h, S, M_1, M_2, R\right) \tag{13}$$

where:

- H = ice thickness,
- Σt°_{-} = sum of negative air temperatures from the beginning of ice drifting,
- Δt°_{+} = influence of thaw weather,
- h = depth of snow cover and ice,
- S = stage of ice formation,
- M_1 = geographical location of water body,
- M_2 = morphological peculiarities of the region of ice formation,
- R = additional local peculiarities of ice cover formation (water power of river, conditions of underground water supply, etc.) and heat inflow to lower surface of the ice cover.

Empirical solutions of these relations were obtained for rivers of European USSR. They are presented in detail by G. P. Bregman in "Manual for forecasting of ice thickness in rivers and lakes".

Freeze-up of rivers results from freezing together of ice fields of different thicknesses. Thus, the initial thickness of the ice cover is heterogeneous. This heterogeneity diminishes during winter due to different thicknesses of snow cover over ice, and to more intensive growth of these ice sections. The heterogeneity of the ice cover is very important for determining the bearing capacity of the ice cover, especially during the first half of winter. The minimum thickness peculiar to the river section must be taken into consideration for calculations.

Growth of the ice cover at the surface occurs when water covers the ice, originating from ice jams or heavy snow loads on the ice cover, and also from increased river run-off. The water layer over the ice can

reach, in some cases, several tens of cm. Water freezing over the ice forms a turbid, whitish snow ice. One or several water layers may be located between surfaces of snow ice and crystalline ice. The thickness of snow ice may reach the thickness of the lower crystalline part. It is necessary to consider, for calculating the bearing capacity of a multi-layered ice cover, that load action is received by the upper layers. The strength of snow ice is less than the strength of crystalline ice.

Some sections of the river surface remain ice-free. These ice-free spaces are known as polynya or mayna. There are two causes of polynya; the thermal, when ice cannot form due to heat inflow from the lower water layers (underground water, sewage, etc.) or the hydraulic, when the speed of flow is high.

Ice melting in the spring occurs on both sides of the cover. Destruction of the ice cover from the upper layers commences after a partial melting away of the snow cover. Ice under the influence of solar radiation decomposes into separate crystals and loses its strength. Warm snow meltwater forming over the ice is also an important factor for ice cover destruction. Snow meltwater coming from bank slopes decomposes the near-shore ice cover. Lower ice strength near banks and an increase of the water level in the river results in shore-clearings. Following, a diminishing of the ice cover strength and an increase in the water level result in breakup of the cover and ice flow.

An accumulation of broken ice in river bends, (ice jams) cause considerable increase in the water level and flooding of banks.

Lakes. Ice appears first in parts of lakes sheltered from waves. Freeze-up occurs simultaneously over the entire surface of the sheltered area. Wind and waves in open areas of lakes cause water mixing, consequently the surface water layers reach the freezing point later than in sheltered areas; on the other hand, they prevent the freezing together of ice fields which promotes floating or drift ice. The increase in the amount of ice occurring during ice formation processes, diminishes the waves on the

water surface, and makes possible the freezing together of ice fields and the formation of a stationary ice cover. If the ice fields are not frozen together, a reversal in the wind direction may produce leads.

Wind and large temperature variations may produce a great stress on the ice cover of large lake sections. This stress will be discussed further. It can be noted now that these stresses result in the appearance of wide fissures, usually in the same places in the lake.

Seas. Formation of an ice cover under conditions in the sea commences with the appearance of slush, islands of which freeze together and produce at low temperatures a thin glass-like crust 5-7 cm. thick, (nylas). Light swell and ripples prevent formation of nylas and then congealed slush forms pancake ice. Slush accumulations near shore freeze to land ice up to several meters wide. Further extension of land ice transforms it into a land floe. Then new unhardened ice is easily fractured into fragments several cm. in diameter, (sludge). Thickening of nylas, and freezing together of pancake ice and sludge form young ice, an even ice layer 7-10 cm. thick, which in time reaches 0.70 m. in seas of the USSR except in polar regions where the thickness reaches 2-3 m.

If the ice cover is exposed to action of the wind or current, then over the even ice cover may form compression ridges or ice piles, (hummocks). A decrease in compression may result in fissures, which on expanding, produce polynya and leads. The fissures frequently cause the movement of large ice fields in the open sea.

The ice-edge seldom has a well-defined boundary. Swell and wind destroy the ice cover and produce sludge near the ice-edge. The pressure of drifting ice on the ice-edge forms rafted ice, when one cake overrides another. This phenomenon may produce double and even triple layers of ice.

Conventional symbols for ice formations in rivers and seas are suggested in Fig. 1.

2. Physico-mechanical Properties of Ice and Snow.

Physical constants of numerous ice formations in water bodies, especially in seas, are not well established. Mechanical properties of natural ice have also been insufficiently investigated. Some physico-mechanical characteristics of ice are suggested.

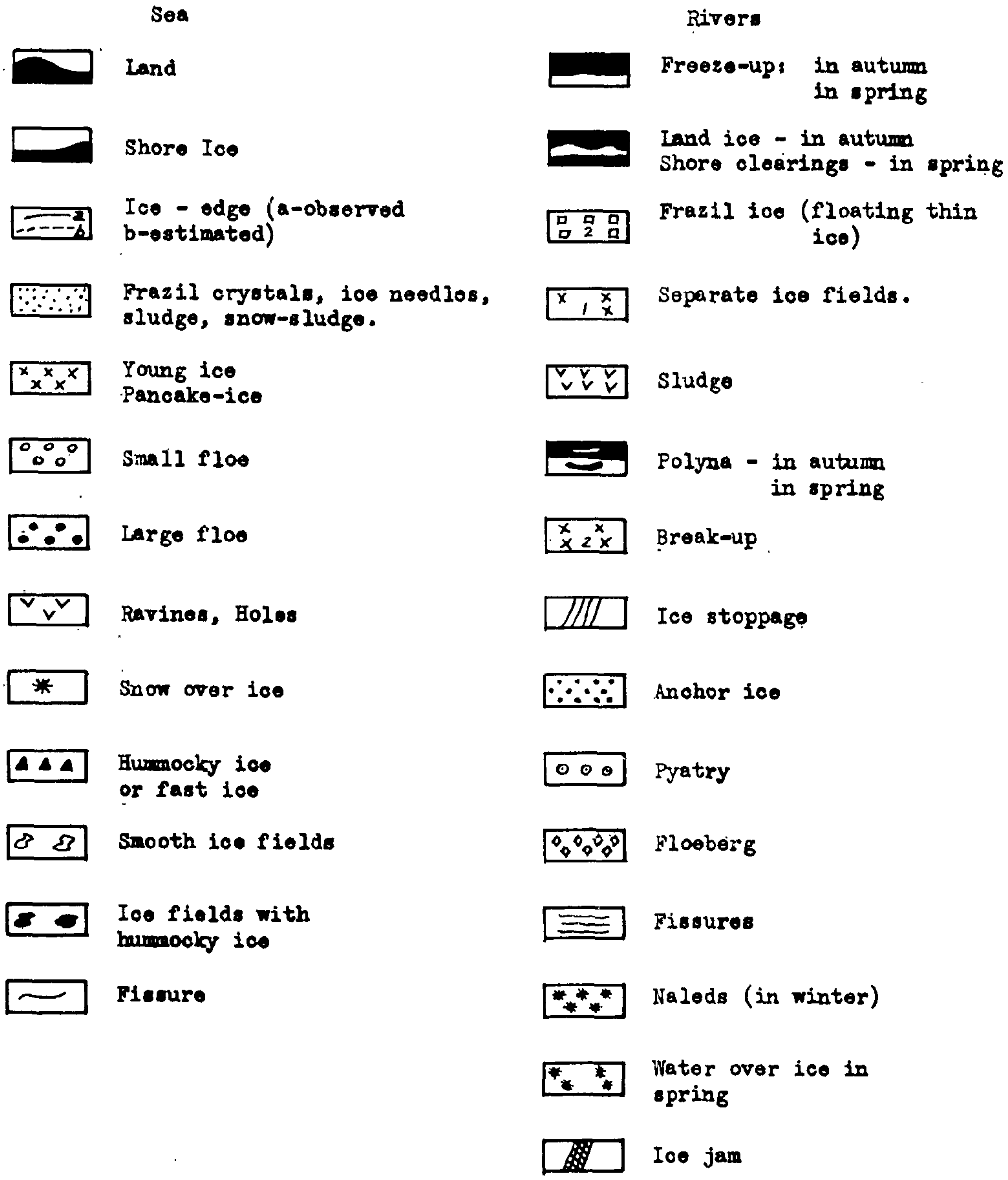

Figure 1. Conventional symbols for ice formations in rivers and seas.

a. Physical characteristics of ice.

Heat capacity of ice. The heat capacity is the heat required to warm 1 kg. of mass from 0° to 1°.* The value for ice varies depending on the temperature:

At ice temperature θ = 0° from 0.4873 to 0.5057

θ = -10° from 0.4770 to 0.4871

θ = -50° from 0.4160 to 0.4156

Dickinson and Osborne proposed the following formula for calculating the heat capacity of ice (C):

$$C = 0.5057 + 0.001863\,\theta.$$

Data on heat required for melting 1 gram of ice at temperature θ ° and salinity S°/oo might be used for sea ice. The following table expresses these values according to Malmgren and Zubov:

Table 3

θ	S°/oo			
	0	5	10	15
- 1	80	60	38	17
- 2	81	70	59	47
- 5	83	77	72	67
-10	85	83	79	76
-20	90	88	86	84

Heat of fusion. The transformation of 1 kg.** of pure ice into water requires, according to Dickinson and Osborne, 79.75 cal. at 20°.***

Editorial Notes: * The values quoted are specific heat values, indicating that this definition is in error and should be a definition of specific heat.

** As stated in original Russian. Should read 1 gram.

*** Apparently means in terms of the 20°C Calorie.

The heat of fusion for sea ice, according to Petterson:

S °/oo	3.02	0.25	4.9	12.0	34.0
L	76.6	77.4	73.7	69.2	55.7

Density of pure ice according to Brosco equals 0.9165 to 0.9174. Density of sea ice varies from 0.936 to 0.827.

Coefficients of expansion and compression of the ice. The increase in unit length or volume/degree is called the expansion coefficient of the solid. It may be characterized by the following values:

The coefficient of linear expansion along the freezing surface varies from 2.84×10^{-5} to 7.36×10^{-5}; the coefficient of the volumetric expansion is 8.1×10^{-5} to 16.2×10^{-5}; the coefficient of ice compression (according to data of Weinberg) varies from 3.3×10^{-5} to 5.0×10^{-5}.

The volumetric expansion of sea ice, according to observations of Petterson, depends largely on ice salinity and temperature. For example, at a salinity of 0.45°/oo and $\theta = -1°$, the coefficient of expansion equals 29×10^{-5}, but at $\theta = -16.2°$ it is 16×10^{-5}; at the same temperatures and a salinity of 6.69°/oo, the coefficient of volumetric expansion equals -389×10^{-5} and + 0.0000 respectively, and lastly, at a salinity of 11.72°/oo it is -128×10^{-5} and -5×10^{-5}.*

Coefficient of thermal conductivity of ice (K_{ℓ}). The coefficient of thermal conductivity is the quantity of heat conducted in 1 second through 1 sq. cm. at a thermal gradient of 1°. According to data of many investigators, it varies near the melting point between 0.0024 and 0.0055 cal/cm./sec./ degree. Variations of the coefficient K_{ℓ} depending on temperature (θ) may be expressed by the formula:

$$K_{\ell} = 0.0053 \quad (1 + 0.0015\,\theta)$$

---

*Editorial Notes: The values quoted should not be used until checked with Petterson's original data as they do not check with Malmgren's quotation of Petterson's values. (Malmgren, Finn., On the Properties of Sea Ice; John Griegs Boktrykkeri, Bergen, 1927)

The thermal conductivity of sea ice has been investigated by Malmgren. The investigations showed that the coefficient for surface sea ice is near the lower value obtained for fresh ice, and at a depth of 1.5 m. near the highest value, i.e., 0.0051.

b. Physical characteristics of the snow.

Density of snow is characterized by the following values:

Dry fresh snow	0.10 - 0.13
Dry compacted snow	0.15 - 0.38
Wet snow, fresh firn	0.32 - 0.38
Wet firn snow	0.41 - 0.48

Abel's, conside ing the heat capacity of snow equal to the heat capacity of ice, suggested the following equation for thermal conductivity of snow:

$$K = 0.0068\ p^2 \text{ cal./sq. cm./second,}$$

where p is snow density.

c. Some physical characteristics of supercooled water.

Density of supercooled water at θ= from 0° to -12° varies between 0.999882 to 0.99754; the coefficient of thermal conductivity for water depending on the temperature θ° may be expressed by the formula of Iakob:

$$K_v = 0.00132\ (1 + 0.0029\theta)$$

d. Mechanical characteristics of ice.

The mechanical properties of ice determining its supporting power are characterized primarily by the following:

(1) modulus of ice elasticity, i.e., stress under which a body obtains a specific elongation equal to one;

(2) elastic limit of ice, i.e., stress at which body retains its elastic properties;

(3) temporary or destructive bending stress;

(4) Poisson's atio - the ratio of specific lateral contraction (elongation) to specific longitudinal extension (contraction);

(5) shearing modulus, i.e., value when relative shear is equal to one.

The numerical values for the mechanical properties of ice suggested by many investigators vary widely. This arises because the experiments with ice samples were usually carried out without considering the speed of loading and the duration of its action. Furthermore, according to experiments, ice samples have mechanical properties somewhat different from those of an unbroken ice cover.

The use of the data obtained for calculating the load capacity of an ice cover is possible by considering the natural ice conditions at ice crossings: ice structure, temperature and duration of present thermal situation, conditions of ice formation, ice salinity, etc.

This indicates that one must select with care the values of ice characteristics for any given case. Data based on experiments and experience of ice-crossing operation are presented in the following.

Modulus of elasticity. Prof. B. P. Weinberg recommended for fresh ice a value for the modulus of elasticity between 70,000 - 80,000 kg./sq. cm. These values are applicable in cases where the loading rate does not cause plastic deformation, i.e., it is less than the limit of ice elasticity. These values cannot be used as calculated for cases of load transportation over an ice cover when the bending stress is considerably more than the limit of elasticity. Lower values of the modulus of elasticity suggested graphically (Fig. 2), obtained on the basis of experiments in the Naval Academy and the Science Research Institute of Hydrotechnology, might be recommended for calculating ice crossings. The graph has 2 parts. In the lower part, the specific load, which is the ratio of load weight to ice thickness, and the duration of load action are considered.

The former value is given along the ordinate, the latter is indicated along any curve. In the upper part of the graph, a system of lines for different ice temperatures are plotted. Calculations of ice temperature were made according to coefficients suggested by B. P. Weinberg for reducing values of bending stress to a given level of temperature. Thus, it was established that the relative variation of the modulus of elasticity

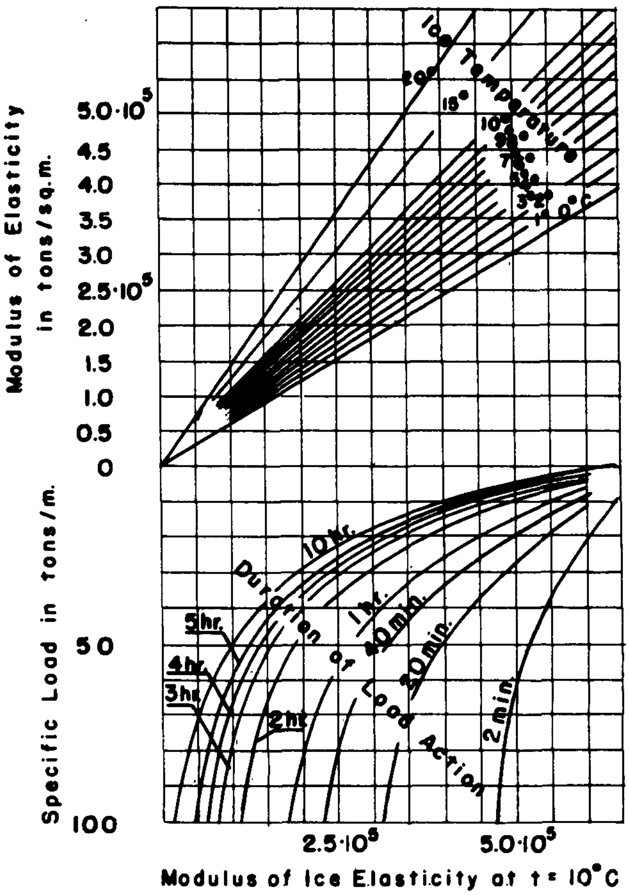

Figure 2. Variations of the modulus of Ice Elasticity (from B.V. Proskurîaov, N.N. Petrunichev, V.P. Berdennikov)

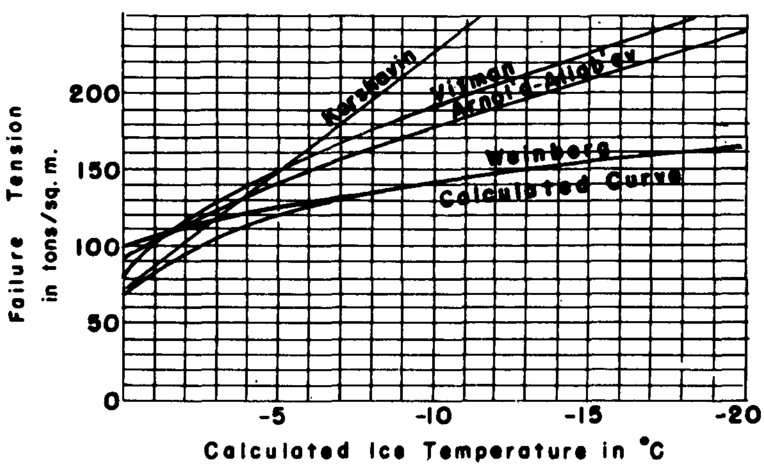

Figure 3. Curves for calculating fresh ice strength

with temperature is the same as the variation of bending stress. This solution is approximate but is the only possible one because experimental data on the dependence of modulus of elasticity on ice temperature are unavailable. However, the dependence of the bending stress of ice on temperature has been studied by many investigators.

The value E may be determined from graphs as follows. The specific loading q is calculated first from the given loading rate Q and ice thickness H by the formula $q = \frac{Q}{h}$ tons/m. Then, the point intersection of the calculated value of specific load with the corresponding curve of duration of load action must be determined. The vertical line from this point upward to the line of given ice temperature will show the value of modulus E for fresh ice.

The lower part of the graph takes into account the action of a standing load, the duration of which is shown along the curves.

If the temperature $\theta = 0^{\circ}C$, then the values of E obtained from the graph are applied for cases when this temperature influences the ice for not more than 1-2 days. The calculation of the modulus for cases of extended warming must be made by the following formula:

$$E = (1 - an) E_o, \tag{14}$$

where:

- a = decrease of modulus of elasticity during thaw weather in kg./cm./day;
- n = number of days from date of appearance of water over ice;
- E_o = value of modulus of elasticity determined from graph for $\theta = 0^{\circ}C$.

It is possible, as a first approximation, to consider a = 0.05 - 0.10, using the first value for cases when the air temperature during thaw weather is well above $0^{\circ}C$ and with considerable cloudiness (little sunshine), and the second value for cases of intensive warming.

Sea ice with lower strength and higher elasticity should have a smaller modulus than fresh ice. It is possible to take this into account as follows, since good experimental data are not available:

$$E_{salt} = (1-0.1S)\,E, \qquad (15)$$

where:

S = salinity, p.p.m. ($^\circ/oo$)

E = modulus of elasticity for fresh ice, determined as shown above.

The formula is applicable for $S \leqslant 5\text{-}6\ ^\circ/oo$.

With a given ice temperature equal to θ°, the determination of E from the graph must be made along the curve for a temperature equal to:

$$t_{salt} = \theta - t_o, \qquad (16)$$

where:

t_o = temperature of water freezing with the salinity of ice. This temperature may be found in Table 4.

Table 4

Temperature of salt water freezing

Salinity S in $^\circ/oo$	0.5	1.0	1.5	2.0	3.0	4.0	5.0	10.0
Freezing temperature	-0.03	-0.06	-0.08	-0.11	-0.16	-0.21	-0.27	- 0.53

Data on ice salinity may be obtained from the offices of the Hydrometeorological Service, or Navy (flotilla, district).

Bending stress of ice. Prof. B. P. Weinberg from experiments on fracturing of ice samples, determined the mean value of the bending stress at $\theta = -3^\circ C$ equal to $\sigma = 16.2 + 4.4$ kg./sq.cm.; is the probable error of the calculated mean value or $\sigma_{max} = 16.2$ kg./sq.cm.

Considering this probable error, the calculated value σ at $\theta = -3°C$ should be:

$$\sigma_{-3} = 11.8 \text{ kg./sq.cm.}$$

Calues of the coefficient $K = \frac{\sigma_{-3}}{\sigma_{\theta}}$ (where σ_{θ} is the bending stress of ice at temperature θ, and σ_{-3}, the same at temperature $-3°C$) suggested in Weinberg's book "Ice" may be used to account for the temperature influence on σ. The value σ_m is calculated for any temperature according to the coefficient K for various ice temperatures and the value $\sigma_{-3} = 11.8$ kg./ sq. cm. Results of these calculations obtained by various investigations are shown in Fig. 3.

The tendency to have a safety factor in calculating ice-crossings necessitates the use of the lower curve for calculating σ max. In cases of important transportation, it is recommended that σ max be determined experimentally, if conditions and time permit.

The relationship shown in Fig. 3, at $\theta = 0°C$, σ max = 7.0 kg./ sq. cm. or 70 tons/sq. m., is valid for the first 1-2 days of thaw weather. If the thaw weather continues, the stress diminishes with ice in a melting state.

Then, according to experiments carried out by M. M. Basin and F. I. Bylov during melting on the Svir River, and by M. M. Basin in Luga Bay, Finland Gulf, it is possible to consider:

$$\sigma_{max} = 7.0 - an. \tag{17}$$

where:

a = decrease of ice stress in kg./sq.cm./day;

n = number of days during thaw weather with water over the ice cover.

M. M. Basin found the following values for shearing strength, a = 0.2 - 0.4 kg./sq.cm./day and for compressive strength a = 1.0 - 2.0 kg./ sq.cm./day. Because the absolute value of bending strength is less than the value of compressive strength and more than the shearing strength, the

calculation of σ_{max} during thaw weather might be made, depending on the intensity of warming by a = 0.4 - 0.6 kg./sq.cm./day.

Consequently:

$$\sigma_{max} = \text{from } (7.0 - 0.4n) \text{ to } (7.0 - 0.6n) \text{ kg./sq.cm.} \quad (18)$$

The strength of sea ice is less than that of fresh ice. Occasional data suggests the following approximate relation for sea ice:

$$\sigma_{t\ salt} = (1 - 0.1\,S)\,\sigma_m, \quad (19)$$

where:

S = ice salinity in $^o/oo$;

σ_m = bending stress of fresh ice, determined by calculated curve in Fig. 3.

This formula is valid for $S \leqslant 5.6^o/oo$.

M. M. Korunov considered the bending stress of sea ice 2-3 times less than for fresh ice. Formula (19) indicates half the bending stress at a salinity of about $5^o/oo$.

Poisson's ratio. B. P. Weinberg obtained experimentally the value of Poisson's ratio for fresh ice $\eta = 0.36 \pm 0.13$, where 0.13 is the probable error of the mean value $\eta = 0.36$. The recommended value for calculation $\eta = 0.33 - 0.25$ or $m = \frac{1}{\eta} = 3-4$.

The shear modulus according to B. P. Weinberg ranges from 20,000 - 30,000 kg./sq.cm.

The compressive strength of ice, according to numerous experiments (by B. P. Weinberg, for the upper ice layers at $\theta = -3°C$) and compressive stress parallel to the crystal axis:

σ parallel = 33 kg./sq.cm.

and with compressive stress perpendicular to crystal axis: σ perpendicular = 25 kg./sq. cm.

and last, for the lower layers:

σ parallel = 31 kg./sq. cm. σ perpendicular = 20 kg./sq. cm.

The tensile strength of ice = 11.1 kg./sq.cm.

shearing strength = 5.8 kg./sq.cm.

torsion strength = 5.1 kg./sq.cm.

These values are given for cases of standing loads.

Ice thickness and temperature distribution in the ice cover are also values which necessitate calculations in addition to the data listed above.

Rating ice thickness. An ice cover formed by freezing from below is transparent and usually has a homogeneous structure. Its mechanical properties are similar at various depths if the temperature conditions are similar. The upper layer of ice is less transparent when freezing occurs not only from below, but also water freezes at the top. Considering that the upper layers of ice work on compression, and that the compressive strength of ice is always many times that of the tensile strength, it is necessary to use as a rating value, the common minimum thickness of the ice and snow ice measured on the ice-crossing route. Cases often occur when water over the ice cover freezes incompletely, forming some water layers in between. Then the thickness of ice without these layers must be considered.

Temperature of ice cover. The temperature of the ice cover varies continuously according to meteorological conditions in the upper layers. It is possible to simplify the calculations by using the mean temperature for a period of similar temperature variations. The change of ice temperatures might be considered as linear from $0^{\circ}C$ in the lower ice surface to the mean air temperature near the upper surface.

The bearing capacity of an ice cover is determined by the strength of its lower layer. The mean temperature of this layer without a snow cover is

$$t = 0.25\,\theta_{mean}. \qquad (20)$$

if the temperature of the upper ice layer is θ mean, where θ mean is the average air temperature for a time interval.

In the presence of a snow cover of $h_{cm.}$ depth, the following approximation can be used:

$$t = \frac{0.06\theta H}{2h_{cm.} + 0.25H} \qquad (21)$$

The coefficients of thermal conductivity in this formula used for ice:

λ ice = 2.0 kcal./m./hr./$^{\circ}$C, and for snow,

λ snow = 0.25 kcal./m./hr./$^{\circ}$C.

If the water body has a salinity S$^{\circ}$/oo, the ice temperature without a snow cover is calculated by the following formula:

$$t = 0.25\theta_{mean} + 0.75\ t_o, \qquad (22)$$

and with a snow cover by the formula:

$$t = \frac{0.25\ H}{2h_{snow} + 0.25H}\ (0.25\theta_{mean} + 0.75\ t_o), \qquad (23)$$

where t_o is the temperature of the water with salinity S $^{\circ}$/oo.

The temperature of the air and the ice thickness for rating period is determined from forecasts issued by the Hydrometeorological Service of the Red Army.

Chapter 5

NATURAL ICE BRIDGES. REINFORCEMENT OF THE ICE COVER

Ice crossings may be classified according to their method of construction into: a) natural, b) reinforced and c) crossings with superstructure.

1. Natural ice bridges.

Natural ice bridges are the simplest and require: a) regular observation of ice conditions, b) the systematic removal of friable snow, which slows traffic and ice growth, and c) the improvement of approaches and exits. A safe crossing during snow storms requires markers along the roadway during the day and lanterns at night.

When snow is removed from the ice, a thin layer of dense snow (about 6-10 cm.) should be left to aid the movement of traffic and prevent damage to the ice surface.

Natural ice bridges may be used not only by sleds or motor vehicles, but even by tanks and artillery, provided that the ice cover is sufficiently thick and air temperatures are low and steady. For the transportation of heavy loads special transport apparatus or reinforcement of the ice crossing may be necessary.

2. Reinforcement of the ice cover.

Ice crossings can be strengthened up to a certain limit by increasing the thickness of the ice cover with artificial ice layers. Artificial ice layers are used with or without other reinforcement.

The complete removal of snow from the roadway is necessary in both cases. The cleared area is covered with a 5 to 10 cm. layer of crushed ice and is flooded with water from buckets, pumps or fire engines. Snow may be used instead of crushed ice. In this case the snow is not completely removed, and the snow layer (3-4 cm. thick) is flooded and frozen. After this layer is frozen, it is covered by a second such layer, etc., until the ice has reached a predetermined thickness and profile. To obtain the right road profile, flooding and freezing should be done from the middle of the road to the sides. It should be remembered that local thickening does not increase the bearing strength of the ice cover. The new ice should be evenly distributed along the whole crossing.

Additional layers can be frozen on the ice crossing at temperatures below $-10^{o}C$ when snow is used, and at $-5^{o}C$ or below if crushed ice is used. Occasionally it becomes necessary to raise the level of the roadway above the ice surface, e.g., near river banks, bridges, etc. In these cases brushwood is used to distribute the load over a large area.

Layers of brushwood or dry branches 3-4 cm. thick (straw is not recommended) are placed on the ice, covered with snow and flooded. It is important that separate layers freeze together well. For this purpose the layer should be lightly tamped. Although the ice produced under these conditions has horizontal stratifications, its strength is close to the strength of natural ice.

The duration of the ice-thickening process is the same for both methods (excluding time spent in transporting brushwood). Two engineer squads using a single hand pump can make an ice layer 100 m. long and 10-15 cm. thick in 2-3 hours.

When necessary, for very important ice crossings, metal screens or cables can be used as reinforcements. The available literature does not show that this method has ever been actually used in practice, but many authors recommended it.

The deflection curve of ice within the elastic limit is expressed by a complicated function. Maximum bending moment occurs in ice under at points where a breaking should be first expected.* The lower part of the ice cover is under tensile stress, the upper under compressive force. Because of the low tensile strength of ice, the lower layer should be strengthened by freezing a metal reinforcement into the ice to distribute the tensile stress over a larger area.

Theoretical calculation of the strength of reinforced ice is difficult because the strength of the metal-ice bond is unknown. However, one may assume that the screen or cable carries a part of the tensile stress and thus diminishes the stress on the ice cover. Used cable, wire (barbed or not), etc. may be utilized for this purpose.

It is easier to prepare the wire screen in small sections 2.0x2.0 m. long and then fasten them together. The interstices should be 1 m.x1 m. in cable mesh and not more than o.5x0.5 m. in wire mesh. To strengthen the reinforcement, poles are fastened to the wire screens and frozen into the ice.

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*Editor's note: This sentence printed as translated, but meaning not clear

Good results are obtained with this method, as was shown during the defense of Leningrad in the winter of 1941-42, where one ice crossing was reinforced. A tank crossing the ice caused it to break near a shell crater. Cables frozen into the ice along and across the ice crossing prevented the tank from sinking too quickly, thus enabling the driver to escape.

Artificial ice layers with or without reinforcement are in most cases the easiest means of strengthening ice especially during periods of thin ice cover. However, there is a limit to the amount of artificial ice that can be used; after this limit has been reached, the lower ice surface starts to melt.

The interrelation between ice thickness at a fixed temperature and heat exchange between the lower ice surface and the water determines the thickness limit. Theoretical calculations show that it is not necessary to determine the thickness limit of the additional layer if the mean speed of water flow is less than 0.4-0.5 m./sec. The melting of ice from below and mechanical ice destruction become hazards only when speeds are above the indicated values. Because the majority of water currents are slowest in winter, it is not necessary, as a rule, to limit the thickness of artificial layers.

An equation to determine the optimum thickness of additional ice layers at speeds of flow more than o.4-0.5 m./sec. is derived in the paragraphs that follow.

Diminishing ice thickness as a function of speed of flow, without cooling from above may be expressed by the equation:

$$\frac{dh}{d\tau} = -\frac{563v^3\mu}{C^2} \qquad (126)$$

where $\frac{dh}{d\tau}$ = derivative of ice thickness and time

v = speed of flow in m./sec.

μ = coefficient, which may be considered as equal to 1

c = coefficient of Chezy

$$C = \frac{1}{n} R^{1/6} \qquad (127)$$

According to the formula, ice is thinner in rapids, which have higher roughness coefficients and accordingly lower values of C, even at the same speed of flow as in a level stretch of water. There the ice cover is unstable and its use for crossing is dangerous.

Considering that the traffic lane of the ice crossing is kept snow-free, the intensity of ice growth is determined by the following equation (assuming that the temperature of ice formation is equal to 0°):

$$\frac{dh}{d\tau} = \frac{\lambda \theta}{LSH}, \qquad (128)$$

where: θ = temperature of the upper ice surface, considering it, in the first approximation, equal to air temperature

λ = coefficient of heat conductivity of ice

L and S = density and melting point of ice.

If time τ is expressed in days and H in cm. then the coefficient of heat conductivity λ is equal to 492. Let S = 80, and L = 0.9, then:

$$\frac{dh}{d\tau} = 6.83 \frac{\theta}{H} \qquad (129)$$

The combinations of equations (126) and (129) gives:

$$H = \frac{6.83}{563} \frac{\theta c^2}{\alpha v^3} = 0.01213 \frac{\theta c^2}{\alpha v^3} \qquad (130)*$$

The last equation makes it possible to determine the practical ice-thickness limit of the adden frozen layer.

The above relationships are derived under the assumption that the ice surface is free of snow.

When the snow has not been removed, the limit of ice thickness is lowered because the intensity of heat flow through the ice cover diminishes according to the proportion:

$$1 + \frac{\lambda_i}{\lambda_{sn}} \frac{\delta}{H} \text{ (sic)} \qquad (131)$$

where: λ_i and λ_{sn} = coefficients of heat conductivity of ice and snow

δ = snow depth over ice

H = ice thickness

which practically diminishes the diminishes the ice thickness value from 2 to 10 times.

As to the decrease in the bearing capacity of ice produced by the added layers, there are no reliable data in the literature and this problem has been solved only very superficially.

* Editor's note: The "α" in this expression is apparently a missprint and should be "μ".

Major General Khrenov, in a manual of military engineering, proposed the following formula:

$$H = \left[h_1 + 0.5\ (h_2 + h_3)\right] k_1\, k_2\ , \qquad (132)$$

where: H = ice thickness

h_1 = thickness of the transparent ice layer in its natural state

h_2 = thickness of the opaque layer

h_3 = thickness of the additional frozen layer

k_1 = structure coefficient equal to 1 with conchoidal and to 2/3 with needle structure

k_2 = thermal coefficient equal to 1 at air temperatures below the f.p. and to 4/5 at air temperatures above the f.p.

Correction coefficients used in this formula presuppose the presence of a linear relation between the ice temperature and the ice-cover thickness needed , which does not exist in reality. The author assumes that the added frozen layer on the traffic lane is capable of supporting an additional load equal to what natural ice of half this thickness can bear. This proposition was never verified either experimentally or theoretically.

It is obvious that the bearing capacity of ice crossings with added layers does not equal that of a natural ice cover as thick as the ice crossing after additional freezing. The increase in the bearing capacity is determined by the width and thickness of the additional layer. An attempt to determine the degree of bearing-capacity increase is presented below, using approximations.

Let us imagine the traffic lane on the ice crossing, where ice layers are added, as a separate unit. Before additional freezing, a portion of the load was carried by the lateral parts of the ice crossing. Let us assume that even after an additional strip of ice has been added in the center of the road, the load on the lateral part has remained unchanged and the bearing capacity of the ice increases only because of the additional thickness of the layer. Then it is possible to consider this strip as a beam on an elastic foundation.

The increase of the deflection line in cases of even load distribution along the beam's length may be expressed by the following equation:

$$y = \frac{Q}{2K\ell^2}\left[2 - e^{-\beta(\frac{\ell}{2}+x)}\cos\beta(\frac{\ell}{2}+x) - e^{-\beta(\frac{\ell}{2}-x)}\cos\beta(\frac{\ell}{2}-x)\right]. \qquad (133)$$

Taking the first derivative with respect to x, we have:

$$\frac{dy}{dx} = \frac{Q\beta}{2K\ell^2}\left[e^{-\beta(\frac{\ell}{2}+x)}\cos\beta(\frac{\ell}{2}+x)^{*} + e^{-\beta(\frac{\ell}{2}-x)^{**}}\sin\beta(\frac{\ell}{2}+x) - e^{-\beta(\frac{\ell}{2}-x)}\cos\beta(\frac{\ell}{2}-x) - e^{-\beta(\frac{\ell}{2}-x)}\sin\beta(\frac{\ell}{2}-x)\right], \qquad (134)$$

Taking the second derivative with respect to x, we have:

$$\frac{d^2y}{dx^2} = \frac{Q\beta^2}{K\ell^2}\left[- e^{-\beta(\frac{\ell}{2}+x)}\sin\beta(\frac{\ell}{2}+x) - e^{-\beta(\frac{\ell}{2}-x)}\sin\beta(\frac{\ell}{2}-x)\right]. \qquad (135)$$

where: y = value of deflection

x = length

Q = load intensity

$\beta = \sqrt[4]{\frac{K}{4EI}}$

K = road-bed coefficient

The deflection moment has a maximum value in the center of load application and equals:

$$M = EI\,\frac{2Q\beta^2}{K\ell^2}\,e^{-\beta\frac{\ell}{2}}\sin\beta\frac{\ell}{2} \qquad (136)$$

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* Translator's note: Omission in book

** Editor's note: The exponent $-\beta(\frac{\ell}{2} - x)$ should read $-\beta(\frac{\ell}{2} + x)$

Maximum stress is determined, as is known, from the ratio:

$$\sigma = \frac{M}{W} \tag{137*}$$

where: W=beam-stretch moment, equal for a rectangular cross section to:

$$\frac{h^2}{6}$$

Thus:

$$\sigma = \frac{3.47\sqrt{EQ}}{\sqrt{K}\, b^2 \sqrt{h}}\, e^{-\beta \frac{b}{2}} \sin\beta \frac{b}{2} \tag{138}$$

Considering that $\frac{b}{2}$ is the approximate radius of load distribution we have:

$$\sigma = \frac{0.87\sqrt{EQ}}{\sqrt{K}\ r^2 \sqrt{h}}\ e^{-\beta r} \sin\beta r \tag{139}$$

or with reduction to the formula (71) for a plate:

$$Q = \frac{\sigma \sqrt{K}\ r^2 \sqrt{h}}{0.87\sqrt{E}}\ \frac{1}{e^{-\beta r}\ \sin\beta r} \tag{140}$$

If tons and meters are used as units $\sqrt{K}$ equals 1, then:

$$Q = \frac{\sigma r^2 \sqrt{h}}{0.87\sqrt{E}}\ \frac{1}{e^{-\beta r}\ \sin\beta r} \tag{141}$$

In equation (71) the value

$$l = \sqrt[4]{\frac{Eh^3}{12}\ \frac{m^2}{m^2-1}} \tag{142}$$

In equation (141) the value

$$\beta = \sqrt[4]{\frac{12}{Eh^3}} \tag{143*}$$

* See note on following page

*Editorial Note:

Equation (143) gives the value of β as $\sqrt[4]{\frac{12}{Eh^3}}$. This value is used in solving equation (137) and for all equations relating to (137). Tables 11 and 12 are a direct result of equation (137) in its expanded form. It is noted that there is an apparent inconsistency between the value of β as defined immediately following equation (135) and the value defined by equation (143). Equating the two values in question:

$$\sqrt[4]{\frac{K}{4EI}} = \sqrt[4]{\frac{12}{Eh^3}}$$

$$\frac{K}{4EI} = \frac{12}{Eh^3}, \quad I = \frac{h^3}{48} \quad \text{For } K = 1$$

The least moment of inertia possible for a rectangular cross section is $I = \frac{bh^3}{12}$. For the computed value of I above to be valid, the value of b must be 1/4. No such assumption is recorded in the text and no logical reason is apparent for assuming such a relationship. Without clarification of this point, Tables 11 and 12 should be used with caution.

Thus, for brevity, it is possible to say that:

$$\frac{1}{e^{-\beta r} \sin\beta r} = \psi\left(\frac{r}{\ell}\right), * \qquad (144)$$

then: $$Q = \frac{\sigma r^2 \sqrt{h}}{0.87\sqrt{E}} \frac{1}{\psi\left(\frac{r}{\ell}\right)} \qquad (145)$$

with increased ice thickness from h_2 to h_1, the bearing capacity of the ice increases:

$$\Delta Q = \frac{\sigma r^2 \sqrt{h_2}}{0.87\sqrt{E}\,\psi_2\left(\frac{r}{\ell}\right)} - \frac{\sigma r^2 \sqrt{h_1}}{0.87\sqrt{E}\,\psi_1\left(\frac{r}{\ell}\right)} \qquad (146)$$

or the relative increase of the bearing capacity in relation to the initial capacity, according to equation (71) is:

$$\frac{\Delta Q}{Q} = \frac{\dfrac{\sigma r^2\sqrt{h_2}}{0.87\sqrt{E}\,\psi_2\left(\frac{r}{\ell}\right)} - \dfrac{\sigma r^2\sqrt{h_1}}{0.87\sqrt{E}\,\psi_1\left(\frac{r}{\ell}\right)}}{\dfrac{0.707\sigma r^2\sqrt{h_1}}{0.275\sqrt{E}} \quad \dfrac{1}{C_2\left(\frac{r}{\ell}\right)}} =$$

$$= 0.445 C_2\left(\frac{r}{\ell}\right)\left[\frac{1}{\psi 2\left(\frac{r}{\ell}\right)}\sqrt{\frac{h_2}{h_1}} - \frac{1}{\psi 1\left(\frac{r}{\ell}\right)}\right]. \qquad (147)$$

The function $\psi\left(\frac{r}{\ell}\right)$, like $C_2\left(\frac{r}{\ell}\right)$, is a periodic function. Its values are given in Table 12. Index 2, with the functional symbol ψ, shows that the function is determined for h_2 and index = 1 for h_1. The function $C_2\left(\frac{r}{\ell}\right)$ is calculated in equation (147) for ice thickness h_1.

Values of $\psi\left(\frac{r}{\ell}\right)$ and $\dfrac{1}{\psi\left(\frac{r}{\ell}\right)}$ are given in Table 11.

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* In view of the development it appears that this substitution should read $e^{-\beta r} \sin\beta r = \psi\left(\frac{r}{\ell}\right)$

Table 11

$\frac{r}{\ell}$	$\psi(\frac{r}{\ell})$	$\frac{1}{\psi(\frac{r}{\ell})}$	$\frac{r}{\ell}$	** $(\psi)(\frac{r}{\ell})$	$\frac{1}{\psi(\frac{r}{\ell})}$	$\frac{r}{\ell}$	$\psi(\frac{r}{\ell})$	$\frac{1}{\psi(\frac{r}{\ell})}$
0.05	0.0477	20.90	0.50	0.291	3.45	1.0	0.310	3.23
0.10	0.0903	11.00	0.60	0.308	3.25	1.1	0.300	3.33
0.20	0.167	6.00	0.70	0.320	3.13	1.2	0.280	3.57
0.30	0.221	4.54	0.80	0.322	3.11	1.3	0.261	3.80
0.40	0.261	3.83	0.90	0.318	3.15	1.4	0.144	4.12
						1.5	0.225	4.45

According to equation (147) the relative increase in bearing capacity of the crossing depends on the load-distribution radius, or, corresponding to conditions, on the width of the additional ice strip and initial and resulting ice thickness. The values of the relative increase in bearing capacity, expressed in percentage of the initial bearing capacity, are given below (see table 12).

Increments in ice bearing capacity for strip widths other than those given above can be obtained by interpolation. Tables show that adding ice is less effective than was assumed [1]. The increase in the bearing capacity by widening the strip is characterized by a curve which levels off. After reaching a certain width, further increase in width produces only a slight increase in bearing capacity.

** This symbol omitted in original text.

Footnote: 1) When using the tables it should be remembered that the percent of increase is calculated in relation to the bearing capacity of the natural ice cover and load distribution along a radius equal to half the thickness of the added layer.

Table 12

Increase in bearing capacity of the ice cover by additional freezing along the traffic lane

Initial Ice thickness in cm.	Final Ice Thickness									
	10	20	30	40	50	60	70	80	90	100
		Increase in Bearing Capacity of Ice in °/o: Width of Additional Ice Layer 2 m.								
10	0	29	42							
20		0	8	16	25	35				
40				0	2	4	7	9	11	12
60							2	3	4	5
			Width of Additional Ice Layer 4 m.							
10	0	24	52							
20		0	10	21	33	46				
40				0	4	8	12	17	22	27
60						0	3	5	8	10
			Width of Additional Ice Layer 8 m.							
10	0	0	0							
20		0	8	19	31	47				
40				0	7	12	19	25	31	35
60						0	4	7	10	13

The above calculations are given for fresh ice. Sea ice is weaker. On the basis of observations, A. M. Batalin considers the additional ice layer useless for operations during the first and even second day of its formation. During this period it is similar to wet, compact snow into which cramped iron penetrates easily. Only on the third day and later does its strength reach a normal value (in Amur Bay from 3-5 kg/sq. cm.). Added layers of sea ice diminish the strength of lower ice layers because the salt solution seeps through. All this indicates that the above calculations are not applicable to sea ice and the usefulness of added ice becomes doubtful in a number of cases.

The time needed to freeze an additional ice layer of a precalculated thickness may be determined by heat conductivity equations for plate cooling. The amount of heat lost by a plate to ambient air is determined from the following equation:

$$q = \alpha(\theta - t)\tau \times F, \qquad (148)$$

Table 12

Increase in bearing capacity of the ice cover by additional freezing along the traffic lane

Initial Ice thickness in cm.	Final Ice Thickness									
	10	20	30	40	50	60	70	80	90	100
	Increase in Bearing Capacity of Ice in %: Width of Additional Ice Layer 2 m.									
10	0	29	42							
20		0	8	16	25	35				
40				0	2	4	7	9	11	12
60							2	3	4	5
	Width of Additional Ice Layer 4 m.									
10	0	24	52							
20		0	10	21	33	46				
40				0	4	8	12	17	22	27
60						0	3	5	8	10
	Width of Additional Ice Layer 8 m.									
10	0	0	0							
20		0	8	19	31	47				
40				0	7	12	19	25	31	35
60						0	4	7	10	13

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The time needed to freeze an additional ice layer of a precalculated thickness may be determined by heat conductivity equations for plate cooling. The amount of heat lost by a plate to ambient air is determined from the following equation:

$$q = \alpha(\theta - t)\tau \times F, \qquad (148)$$

where: θ = air temperature

t = plate temperature

α = coefficient of heat conductivity

F = cooling surface

τ = time

The amount of heat lost to water poured over the ice may be expressed by the same equation. Considering that the temperature of water poured over ice is 0° and the surface heat loss is accompanied by the liberation of the latent heat of freezing, we have:

$$\alpha \theta \tau = 720H \qquad (149)$$

On the left side of the equation the heat flow is given for 1 sq.m. of surface, H = ice thickness in cm., and α = coefficient of heat conductivity in kg.cal./sq.m./hour °C.

Solving the last equation for H, we get:

$$H = \frac{\alpha \theta \tau}{720} \qquad (150)$$

The coefficient of heat conductivity α, with wind speed taken into account, will be:

$$\alpha = (28 + \theta)\sqrt{v + 0.3}. \qquad (151)$$

Substituting in equation (150)

$$H = \frac{\theta\,(28 + \theta)\,\sqrt{v \text{ to } .3}\,\tau}{720} \qquad (152)$$

This equation is applicable only to a thin layer of water. If the thickness of the water layer is more than a few cm., ice formation is retarded, which is not desirable. Table 13 is compiled according to equation (152).

Table 13

Thickness of ice formed by freezing within 1 hour

Wind Speed	Air Temperature in °C						
	-4	-5	-10	-15	-20	-25	-30
	Thickness of Ice Formed in 1 hr. in cm.						
0	0	0	0.5	1.0	1.5	2.0	2.5
1	0	0	0.5	1.0	1.5	2.0	3.0
3	0	0	1.0	1.5	2.5	3.5	4.5
5	0	0	1.0	2.0	3.0	4.0	5.5
7	0	0.5	1.5	2.5	3.5	5.0	6.5
10	0.5	1.0	1.5	3.0	4.5	6.0	8.0

These values are applicable to fresh water only. Ice formation from salt water depends on more complicated relations. The use of salt water to increase the bearing capacity of the road is not permissible because of its low strength.

The values presented in table 13 agree well, at low wind speeds, with observations made by M. M. Korunov on the speed of additional freezing. Korunov proposed the following formula:

$$\tau = \frac{790H}{\theta} \qquad (153)$$

where τ is expressed in min., H in cm. and θ in °C.

3. Superstructure of ice crossings.

Ice crossings with superstructures are used to prevent the ice surface from being crushed or when it is impossible to increase the thickness of the ice by adding ice (particularly at air temperatures above -10°C).

Protective layers are intended for pedestrian traffic or for wheeled, motorized or caterpillar traffic. A footpath for pedestrian traffic is constructed when there is water over the ice or if there is danger that the ice might break. For temporary crossings, dangerous areas are covered with poles, planks or other wood. For permanent ice crossings it is better to lay planks over beams or short rods. Planks are fastened to cross beams with nails. The interval between beams must be such that plank deflection under human weight is insignificant. The width of the footpath must be sufficient for two-lane pedestrian traffic. Posts are fastened to cross beams on one side of the path. Poles or ropes are used as railings. For illumination at night, lanterns and electric bulbs are suspended on Г form posts.

Similar footpaths are found in cities and villages. Pedestrian crossings in zones of military operations are temporary and permanent paths are seldom built.

Protective roadway surfaces for motorized or caterpillar traffic are not built along the whole stretch, only at approaches to crossings, over fissures, etc. The roadway surface is constructed from planks for motor-vehicle traffic or from slabs for caterpillar traffic and is placed on thick longitudinal poles or beams, frozen into the ice. The spaces between the beams are filled with snow or crushed ice and flooded. Roadway surfaces for wheeled and caterpillar traffic should be constructed from slabs with their flat surface upwards.

Foundation beams are placed close together under the wheel or caterpillar tracks and some distance apart in the middle of the road. For a roadway 4 m. wide, with planks 5 cm. thick and beams 20 cm. in diam., it is sufficient, for car and truck traffic, to space 7 longitudinal beams at the following distances from the outside edge of the roadway:

axis of 1st beam = 0.10 m. from the edge
" " 2nd beam = 0.75 m. " " "
" " 3rd beam = 1.25 m. " " "
" " 4th beam = 2.0 m. " " "
" " 5th beam = 2.75 m. " " "
" " 6th beam = 3.25 m. " " "
" " 7th beam = 3.90 m. " " "

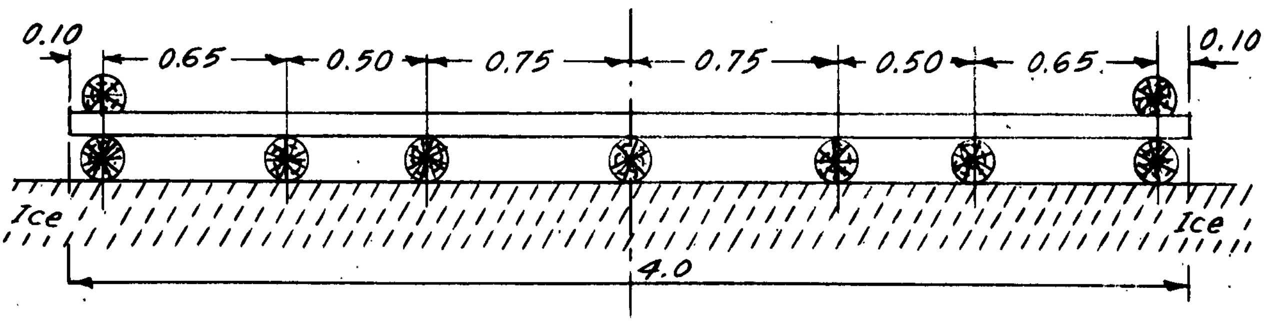
The distribution of beams is shown schematically in fig. 24. The distance between beams should be shortened if thinner planks are used.

The surface material is fastened to the beams with nails, and curbs are added.

Superstructures for the reinforcement of ice crossings must meet the following specifications:

a) They must distribute the load on the ice cover over a precalculated radius.
b) They must have the least weight and have the required hardness*.
c) They must be simple and easy to construct, if possible, from local materials.

Fig. 24, Cross-sectional diagram of the flooring of an ice-crossing superstructure



* Editorial Note: The term "hardness" as used in this chapter, is more correctly expressed as "rigidity".

The radius of load distribution is determined as follows. Let us suppose that, according to specifications, the ice crossing must be calculated for a 30 ton load over a 35 cm. ice cover. Then it is possible to use a constant value for the modulus of elasticity. If we consider this value as being 3×10^5 ton/sq/m. and Poisson's coefficient as 3, then value ℓ, according to formula (69)*, must be:

$$\ell = \sqrt[4]{\frac{3x10^5x0.35^3}{12} \times \frac{9}{8}} = 5.9 \text{ m.}$$

If we turn to formula (71), first of all let us consider the allowable stress. Let us suppose that the indicated loads would be carried at air temperatures not higher than -10°. Thus σ_{max}, according to graph 3, must be 112 ton/sq.m. Substituting into formula (71) the known values, we have:

$$** \quad 40 = \frac{112}{0.275} \; \frac{\sqrt{0.35x0.707r^2}}{\sqrt{3x10^5} \, C_2 \left(\frac{r}{\ell}\right)} \; ; \; \frac{r^2}{C_2\left(\frac{r}{\ell}\right)} = 96.5$$

Approximations are used for further solution. If we assume that r = 5 m., then $\left(\frac{r}{\ell}\right) = 0.843$. According to the table giving C_2 values, $C_2 = 0.295$.

Then:

$$\frac{r^2}{C_2 \left(\frac{r}{\ell}\right)}_{r=5} = 85.8$$

Consequently, the radius of distribution must be greater. Let us suppose that r = 6 m., then:

$$\frac{r^2}{C_2 \left(\frac{r}{\ell}\right)}_{r=6} = 104,$$

i.e., the value is sufficiently close to the required radius of distribution. For r = 5.5 m.:

$$\frac{r^2}{C_2 \left(\frac{r}{\ell}\right)}_{r=5.5} = 94.3$$

Thus r = 5.5 m. is selected for further calculation. The problem now is to determine the hardness of the construction necessary for a required load distribution.

$$* \; \ell = \sqrt[4]{\frac{m^2 \, Eh^3}{12(m^2-1)}}$$

** 40 appears to be an error in printing; should be 30 in order to obtain the results shown.

Load intensity is determined from the equation:

$$q = \frac{Q}{\pi r^2} = \frac{30}{3.14 \times 5.5^2} = 0.317 \text{ ton/sq m.}$$

Then the equation for ice-cover deflection under load must be:

$$Y = 0.317 \quad \left[1 + C_1 Z_1 (\alpha) + C_2 Z_2 (\alpha)\right]$$

Ice Deflection at two points, in the center of load distribution and at a distance of 5.5 m. from it, is determined as follows: The integration constants C_1 and C_2 for $\alpha = \frac{r}{l} \; 0.927$ are, respectively:

$$C_1 = -0.733, \; C_2 = 0.329$$

where: $x = 0$; $Z_1(\alpha) = 1.0$; $Z_2(\alpha) = 0$

where: $x = 5.5$ m.; $Z_1(\alpha) = 0.989$; $Z_2(\alpha) = -0.215$;

$$y_{x=0} = 0.317 \; (1-.733) = 0.085 \text{ m.};$$

$$y_{x=5.5} = 0.317 \; (1-0.725-0.072) = 0.065 \text{ m.}$$

The difference in deflection of the two points is 0.020 or ~ 2 cm.

Considering, approximately, that the hardness of the bridging materials is equal to that of the beam, 5.5 m. of which is fastened, and the load proportionally distributed, we get a maximum deflection of 2 cm. This results from an analysis of the joint action of the superstructure and the ice cover. If the load is located at point A (fig. 25), the ice cover exerts its pressure on the superstructure in an upward direction. In order that the load be distributed proportionally, the ice cover deflection should equal that of the superstructure. The load should be distributed, according to specifications, along a radius not less than 5.5 m., or along a beam 1 m. wide and not less than 11 m. long. Then the load center can be considered as a support at a point and the action of the ice cover as the deflecting force.

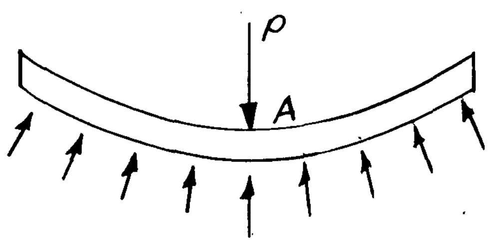


Fig. 25. Scheme of the pressure exerted by the ice cover on the superstructure

** The following equation determines the maximum deflection of a cantilever, loaded unifromly

$$y = \frac{q x^4}{12EI} \text{ (1)*} \qquad (154)$$

Substituting numerical values, we get:

$$2 = \frac{0.0317 \times 5.54}{18EI} \text{ (2) (3)}$$

Thus:

$$EI = 1.81 \times 10^8.$$

If modulus of elasticity of wood E = 100,000 kg./sq.cm., then moment of inertia:

$$I = 1.81 \times 10^2 \text{ (4)}$$

The height of the whole superstructure is determined:

$$1.81 \times 10^3 = \frac{?}{12} \text{ (5)}$$

$$h = 28.0 \text{ cm.}$$

It is sufficient to determine the hardness of the bridging materials in order to select the type of superstructure needed. In the above case, a simple roadway surface from beams is obviously sufficient. Because the specified conditions require an even load distribution along and across the roadway it is necessary to build a double structure with beams along and across the traffic lane.

Thus, girders and foundation beams can be spaced. Girders should be spaced wider in the middle of the road and cloase together under the wheel tracks. Wooden plates or thick planks are used for flooring and curb construction. A typical beam arrangement is shown in fig. 26. It must be remembered that the hardness at any point in the superstructure must not be below the calculated value. Thus, butts should be connected in dovetail fashion and joints should bend easily. Foundation beams should be fastened to girders with bolts, etc. After the construction plan has been worked out, the calculations should be rechecked, taking also into account the weight of the structure.

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* Translator's note: Corrected according to the errata list

** Editorial note: The following is a list of apparent printing errors on this page as shown by circled numbers:

(1) should be 8EI
(2) should be 5.5^4
(3) should be 8
(4) should be 10^3
(5) should be h^3

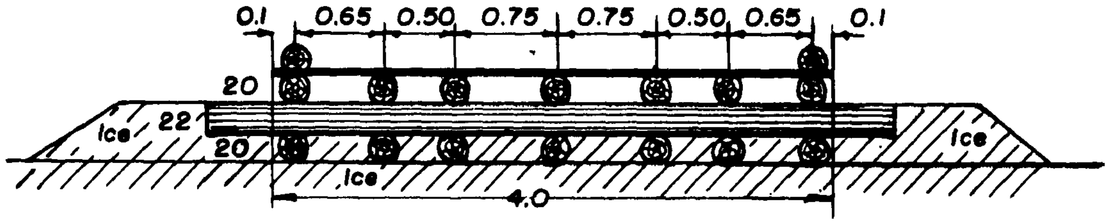

Fig. 26. Cross section of the reinforced superstructure

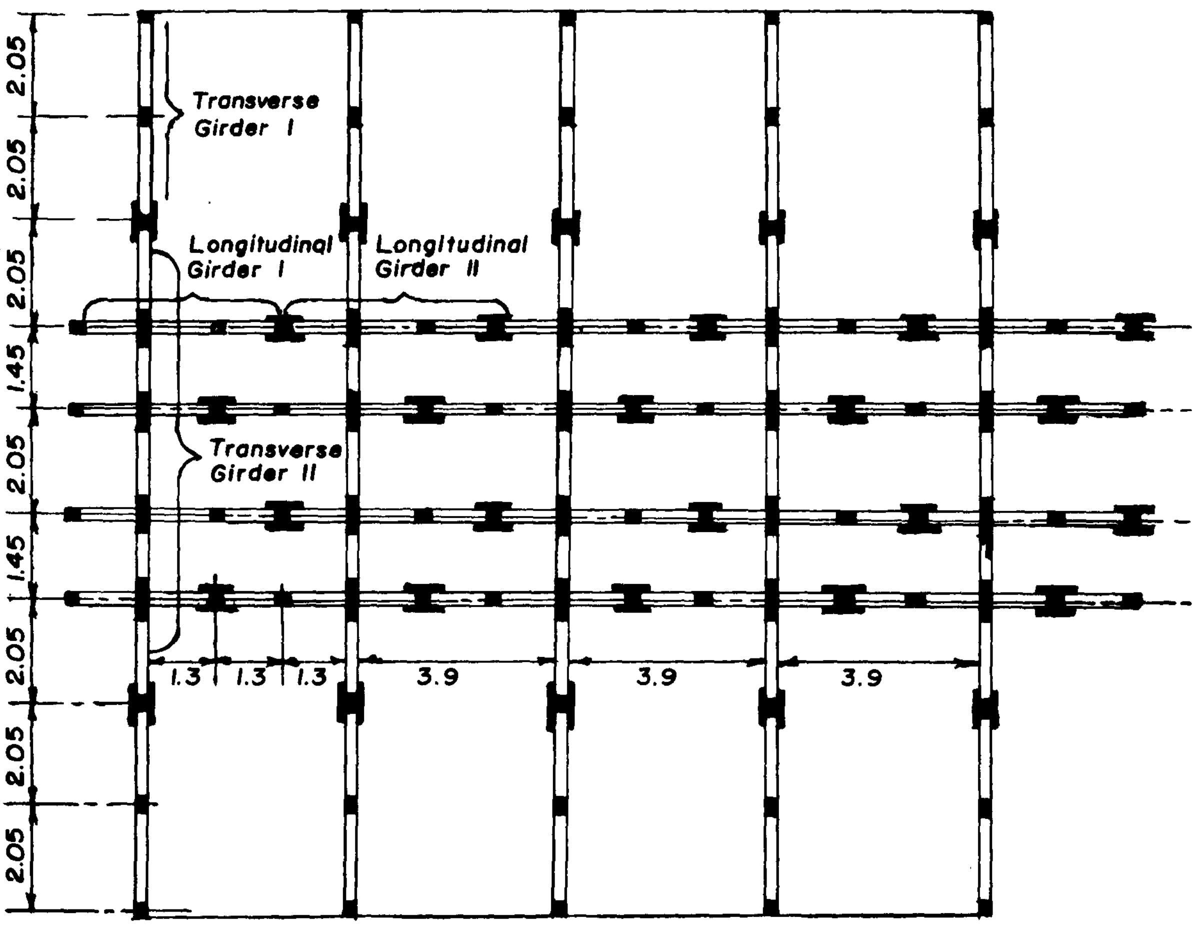

Fig. 27. General scheme of longitudinal and transverse frames.

The above case was based on the assumption that it is possible to distribute the load evenly along and across the entire crossing. The hardness of the structure has been calculated accordingly. However, in some cases, especially if the radius of load distribution is very great, it is difficult or even impossible to reach an even hardness across the road. In this case a decrease in the width of the superstructure would make possible a more even load distribution along the crossing.

The necessary increase in lengthwise load distribution is determined from the following relation:

$$\ell = \frac{\pi r^2}{b}\,.$$

where: ℓ = needed lengthwise load distribution

b = transverse load distribution

r = load distribution radius, determined from formula (71).

In order to find the hardness of the superstructure it is necessary that the deflection of the superstructure and the ice cover be equal at a distance $\frac{\ell}{2}$ and not r. Consequently, in formula (154), x is substituted by $\frac{\ell}{2}$, thus the hardness and the required moment of inertia increase, making it imperative to increase the height of the structure. The use of more than 3 layers of beams is not recommended because then the structure becomes too heavy. Thus, when calculations show that a 3-layer structure is not sufficient it is possible to use a timber frame. To calculate the timber frame deflection an approximate method can be used, recommended by Professor Paton, where the frame is considered as a wall, weakened by apertures. In this case all formulas used for beams are applicable to the timber frame. The moment of inertia of the upper and lower timbers, if cross sections are equal, must be approximately:

$$I = 2F\left(\frac{h}{2}\right)^2 = F\frac{h^2}{2} \qquad (156)$$

where: F = cross section of the timbers

h = height in the center of the frame, determined by the centers of gravity of the timbers.

Here we neglected the moment of inertia of the cross section of the timber in relation to its neutral axis. If cross sections of individual timbers vary,

then the moment of inertia should be considered as the sum of moments of inertia of the upper and lower timbers relative to the height of the frame at the center. The use of timber frames has the advantage that they can be constructed at a distance, transported in sections to the crossing and assembled there. Structures with frames are not as heavy, but costly construction materials like, planks, cross bars, special forged pieces, etc. must be used.

At one of the war fronts, a wood frame was used for tank and railroad traffic. The load was distributed on transverse frames of the same type as the longitudinal frames. A general scheme of longitudinal and transverse frames is shown in Fig. 27, and their separate sections in Fig. 28.

The floor and curbs were laid on top of the longitudinal frames. Beams 20 cm. in diameter were frozen to the ice cover under the frames.

It is better to freeze single or multiple-layer floors into the ice; the upper layer of beams is installed only after the lower layer has frozen. To speed up the freezing process, the spaces between the beams are filled with snow or crushed ice and flooded.

Bridging equipment A-3 or pontoon equipment N-2-P may be used as a last resort when no wood is available in the vicinity.

Two types of superstructure can be made from the bridging equipment (A-3), according to Brigade Engineer Lebedev.

One type of construction is shown in Fig. 29 and the second in Fig. 30.

The installation of the first type of crossing proceeds in the following order. First, the snow is removed from the center of the road, and then foundation beams, coupled with standard parapet supports, are laid. To determine the right pattern for the foundation beams, long coupling rods are used as a standard. Foundation beams are used only for installation and are not included in calculations for the construction.

After the beams have been laid according to the scheme in Figs. 29-30, transverse girders (b) from long and short coupling rods and beams for curbs are added. Girders are placed on the ice with their narrow edges down and dowels up. Then longitudinal girders (V) are installed by inserting their dowels into the holes of the foundation beams.

Then snow is packed under the beams and coupling rods.

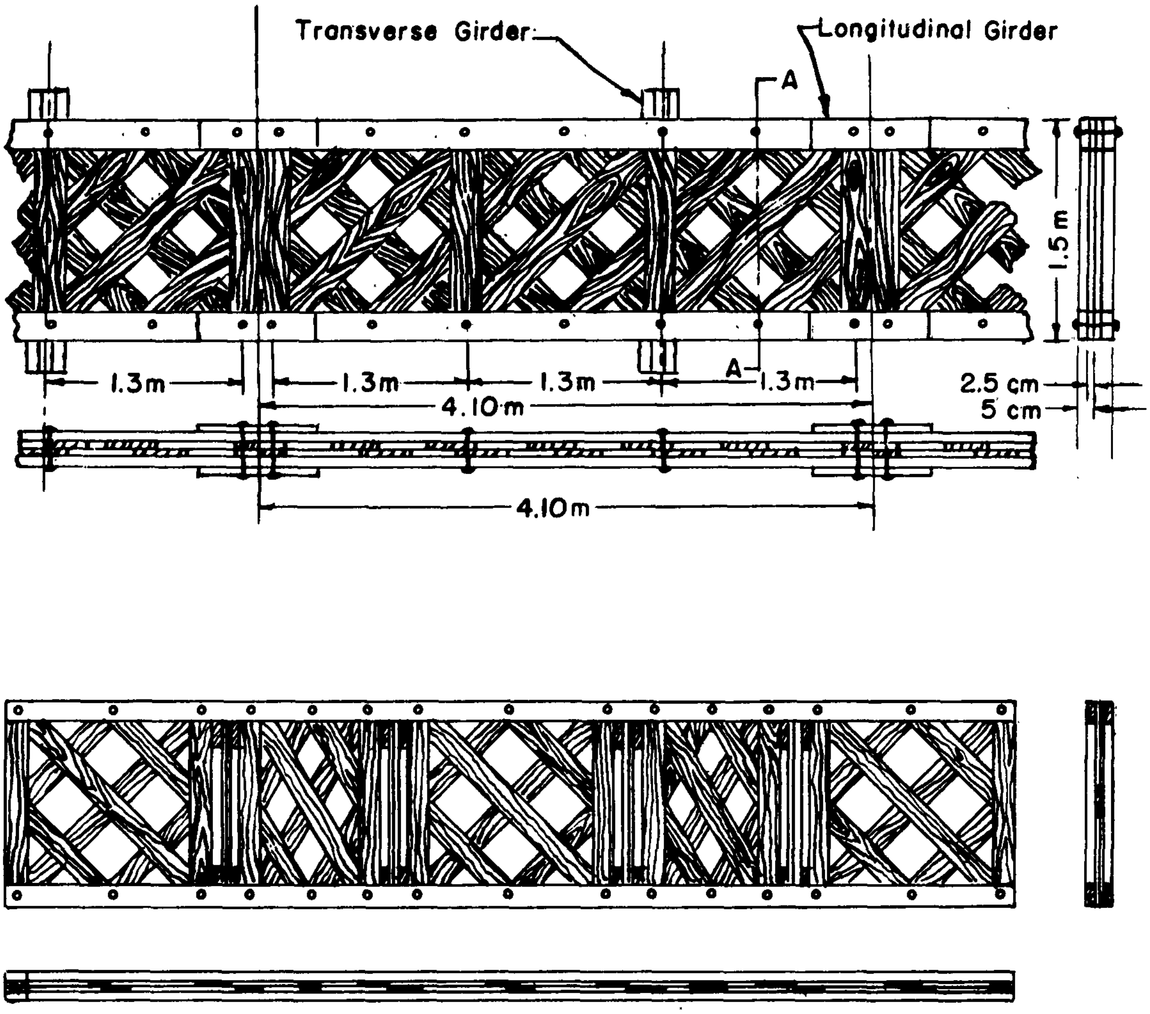

Fig. 28. Longitudinal and transverse sections of the frame.

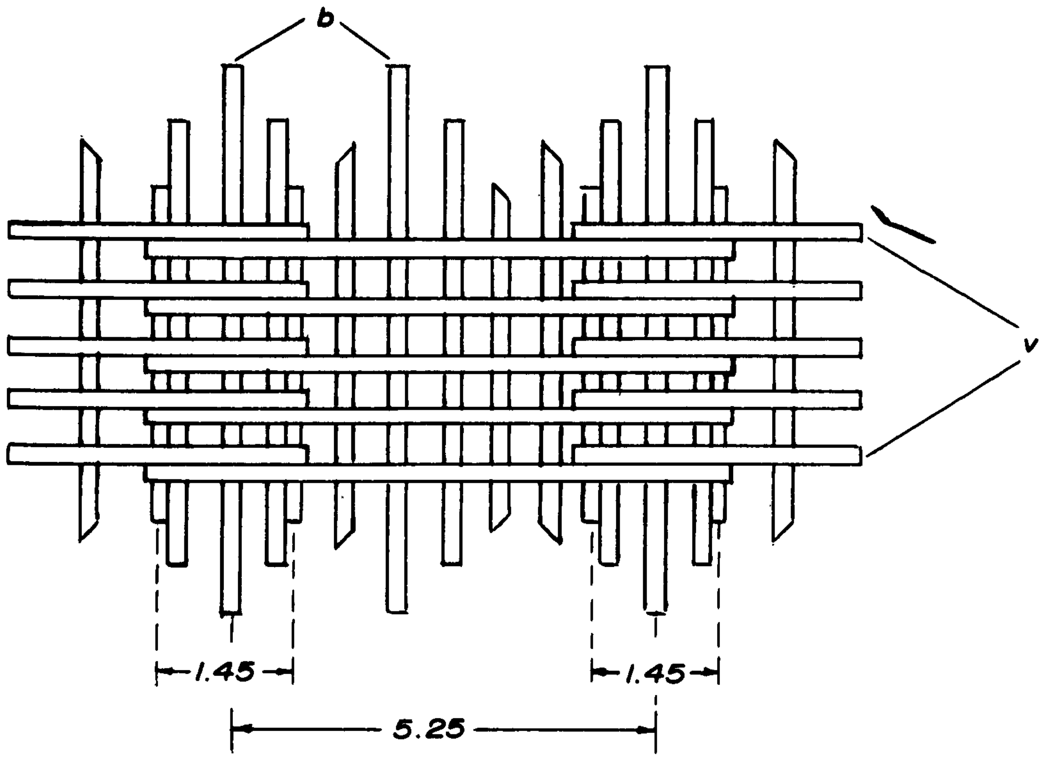

Fig. 29. Scheme of superstructure built from bridging equipment A-3, Type 1.

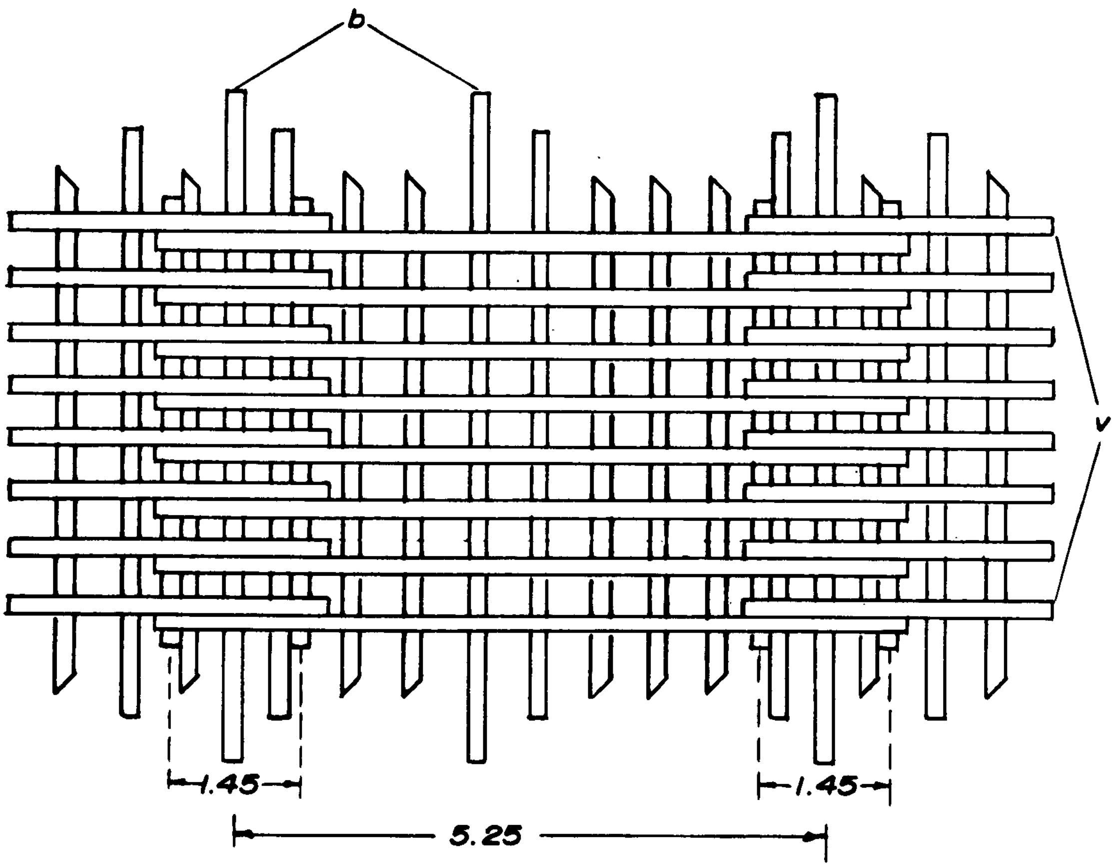

Fig. 30. Scheme of superstructure built from bridging equipment A-3, Type 2.

Because of the instability of the beams, set with their narrow sides down, it is necessary to flood the structure and let the lower half of the beams freeze into the ice. Then the other coupling rods are placed where they belong, the floor is installed and the whole is frozen together.

One set of bridging equipment A-3 is sufficient for 12 sections or 63 running meters of the superstructure.

The installation of the second crossing differs only in the number of cross beams (10 against 8 for one section) and longitudinal coupling rods used (8 against 5). Here it is possible to install, from one set, 9 sections or 47 running meters of the superstructure.

A scheme of the superstructure from pontoon equipment N-2-P is shown in Fig. 31. This structure is installed as follows. The foundation for the superstructure is prepared first. But instead of using transverse half-beams (b) alone, 2 standard panes are placed underneath. Longitudinal half-beams are then installed and fastened together with couplers.

For stability, the structure is reinforced by planks, fastened together with bolts. Two crosswise on each beam and then after the floor has been installed 3 lengthwise on top.

The location of the planks and bolts is shown in Fig. 31, together with the number of standard panels imbedded between cross-beams.

The construction of the floor is similar to the construction of the bridging.

One set of pontoon equipment is enough for 12 sections, or 70 running meters.

4. Specific cases of heavy-load crossings.

If heavy loads are to be transported when the ice is not thick enough, the following should be considered:

a) whether loads are to be transported regularly

b) or the crossing is to be used temporarily.

In the first case the crossing should be built securely, in the second, means should be used which are less time consuming and less expensive, even if they are less safe.

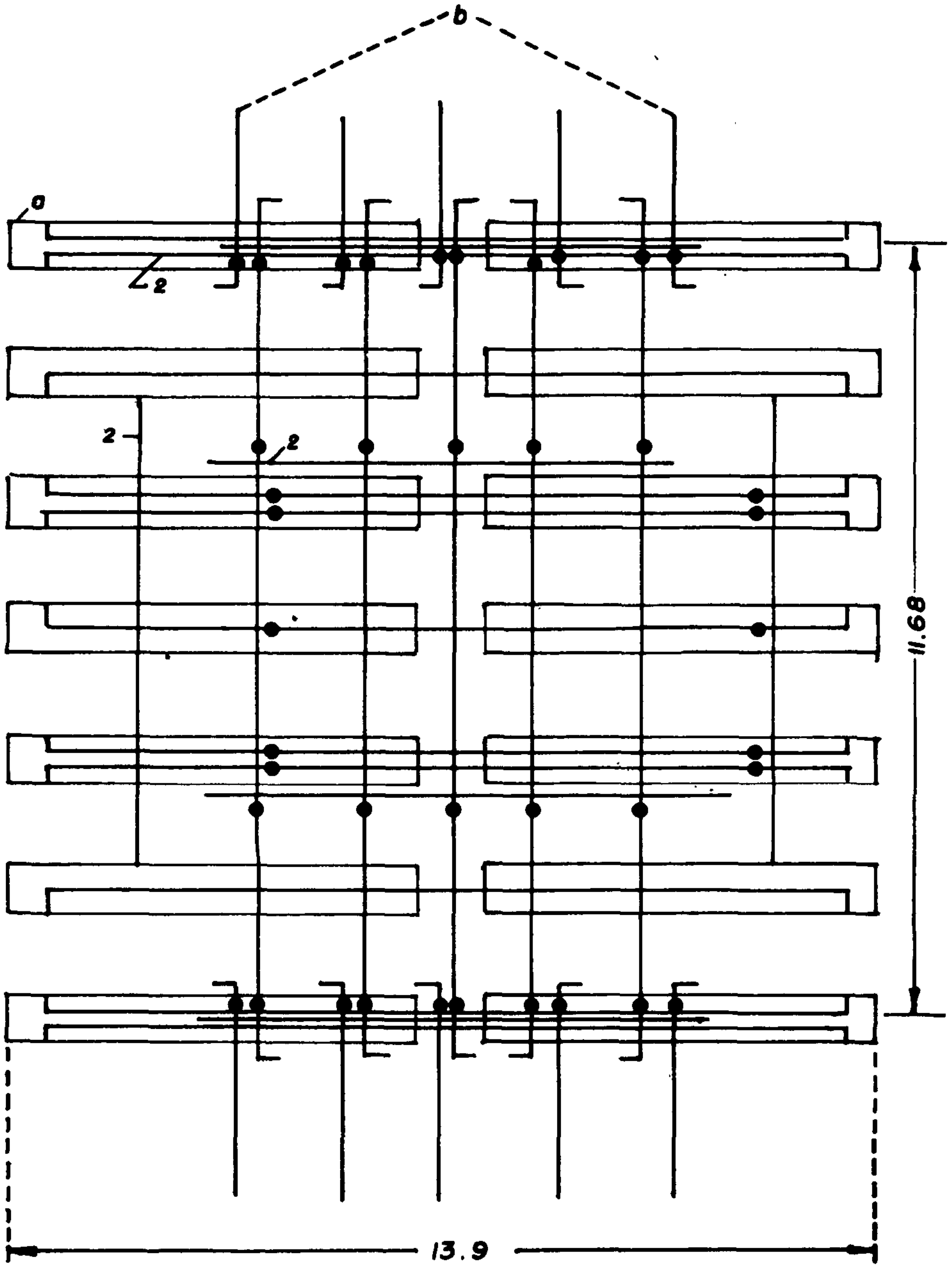

Fig. 31. Scheme of superstructure built from pontoon equipment H-2-P.

When heavy loads are to be transported regularly and it is impossible to reinforce the ice cover by additional freezing or by a superstructure, it is possible to construct a pontoon bridge. For this purpose, wide trenches are cut in the ice and the pontoons emplaced. A flooring of the same pontoon equipment is put over the pontoons.

Crossings are especially good when piles are used along the whole route. A structure of this type can be used only in rivers with small water-level variations, otherwise the ice would lift the piles from the river bottom. Settling of the ice cover may cause a deformation in the crossing. Such a crossing over the Kola River was constructed by A. Ch. Kunitskii for railroad trains.

It was constructed in the following manner. Every 2 m. along the rails, holes were made in the ice and pickets inserted into the holes until they reached the foundation beams, which were 22-26 cm. thick. The pickets were cut off to the ice surface and frozen into it. The remaining space in the holes was filled with crushed ice or snow. Later work showed that this method is superior to wedging for the following reasons. Driving a wedge into the extra space in a hole required 10-15 minutes as against 1-2 minutes for packing, and sometimes pickets were forced out of the hole before they had time to freeze. Furthermore, water rose from under the ice and work had to be done in freezing water. Packing with crushed ice and snow never caused the water to rise.

After the pickets were installed, cross beams 4-4 1/2 m. long and 22-26 cm. in diameter were installed on top in a checkered pattern. (Fig. 32)

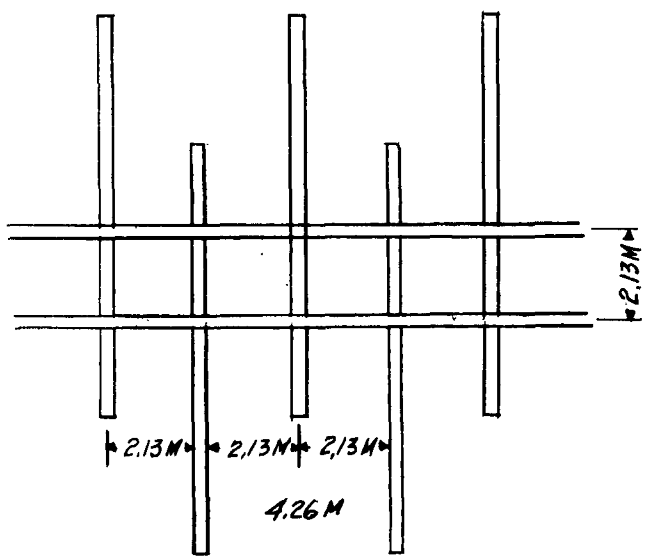

Fig. 32 Scheme of laying cross beams on top of cut off pickets

Longitudinal beams of the same thickness were placed over the cross beams. The space between the ice surface and the upper part of the longitudinal beams was filled with brushwood, packed with snow and flooded. Then the surface was levelled with snow in order to install the sleepers.

After the rails were set on the ties, the space between was packed with snow, flooded and frozen (ties are not shown in the fig.). A beam foundation is only practical with a hard river bed and in rivers with small water-level variations. In rivers with high water-level variations the beam foundation would rise with the ice cover.

For occasional traffic a superstructure is undesirable because of the difficulties and the work involved. At the same time, if the load is not distributed over a large area, it cannot be transported over the ice. In this case sleds may replace the superstructure. Sleds can be moved over the ice by winches installed on the river bank, provided the distance to be crossed is relatively small (up to 500 m.), or by a tractor connected to the sled by a long cable (not less than 50-70 m.).

Sleds should meet the same requirements as a superstructure, namely:

a) load should be distributed according to equation (71);
b) sleds should be as light as possible;
c) runner deflection should equal the deflection of ice;
d) the distance between runners must not be more than 2-3 m. and pressure on each not more than the value of the shearing strength of the ice.

Thus it is possible to use the same methods of hardness selection as were applied to the superstructure. Because the radius of load distribution might be rather large, a frame construction or, in some cases, special materials like multiple-layer plywood, seamless steel piles, etc. may be used in order to get the necessary hardness. To ease the movement of sleds it is necessary to remove the snow from the road or if need be flood it, i.e. build an ice road similar to roads used in the lumber industry.

Occasional traffic should be transported by sleds when air temperatures are lowest with little variations during 3-4 days, because then the bearing capacity of the ice is highest.

The crossing site should be specially selected. The ice along the traffic lane must be homogeneous without hummocks and fissures and be as thick as possible.

If the ice is covered with a thick layer of snow it should be removed shortly before the load is transported to prevent air temperatures from affecting the ice cover.

5. Approaches and Exits

The construction of approaches and exits is an entirely different and difficult problem. Difficult because of water-level variations and consequent changes in ice-cover level, heterogeneity and softness of the ice cover near banks, fissures, glimmer-ice formations, ice sagging, etc.

The construction of approaches to crossings involves: the levelling of approaches and bank slopes and the installation of means for safe load transfer from the bank to the ice.

The conditions required for the construction of approaches are:

a) rectilinearity of the traffic lane and sufficient width, not less than 6 m.;

b) inclination slope of not more than o.2 for caterpillar traffic, o.1 for wheeled traffic and 0.01 for railroad trains (for sled traffic it may be greater);

c) sturdy construction where the approach connects the bank to the ice, with provisions for probably variation in ice-cover level.

The first two specifications may be realized easily by properly selecting the site of the crossing. Large depressions in frozen ground should be avoided.

The construction of the ramp from the bank to the ice depends largely on the type of traffic over the crossing, the hardness of the ice near the bank and water-level variations.

For pedestrian traffic, planks or boards can be used when the distance from the bank is not more than 15-20 m. and the ice-cover is hard enough. For car and truck traffic the ice cover has to be reinforced by additional freezing up to a thickness of 1.0-1.5 m. Then the traffic lane is covered with brushwood, packed with snow and flooded. The surface of the frozen ramp is covered with rods or planks, layed crosswise. Logs are used as curbs and if the ramp is too high, parapets are built.

The greater the water-level variations, the longer must be the ramp. Ice deflection usually begins at 10-15 m. from the bank. Therefore the ramp must not be shorter than 20 m.

An extension of planks on cross beams should be added to the lower end of the ramp to guard against cracks in the ice and to prevent crushing of the ice surface, even if no superstructure is planned. Approaches are marked, and lit by lanterns at night.

For ramps it is sometimes convenient to use river piers, which are usually accessible by good roads. In this case the flooring from the pier to the ice surface is bolted to cross beams. When the water level rises, some of the supporting beams are removed; when it drops, beams are added. A sketch of such a ramp is shown in Fig. 33. When possible, a barge or any other type of vessel may replace the support. Then the ramp is built on permanent supports which do not require changing the number of beams or constant observation of the water level. Ramps from the pier to the barge are easily constructed with the flooring on longitudinal logs. A diagram of this type of installation is shown in Fig. 34.

When the ice is unsafe near the bank or if there is an ice-free gap, it is imperative to transfer the load pressure to the ground. In this case small pile bridges with plank flooring are built for pedestrian traffic. A width of 2-3 planks is sufficient for short bridges.

Scaffold or pile bridges are constructed for motor traffic. Ramps from bridges to the ice are built of beams and planks. One end of the beams is placed on a supporting girder, the other on the bridge. Piles may be replaced by trusses and joists. The height of the truss or scaffold must exceed the maximum possible water level.

In rivers with large water-level variations, more efficient approaches must be constructed. The same approaches are constructed for all railroad crossings. Such approaches must slope smoothly from the bank to the ice under any conditions of water-level, with construction time held to a minimum. When the range of water-level variations is known, the length of the ramp determines the degree of slope. The weight of the load to be carried and the length of the approach determine the planning of the operation.

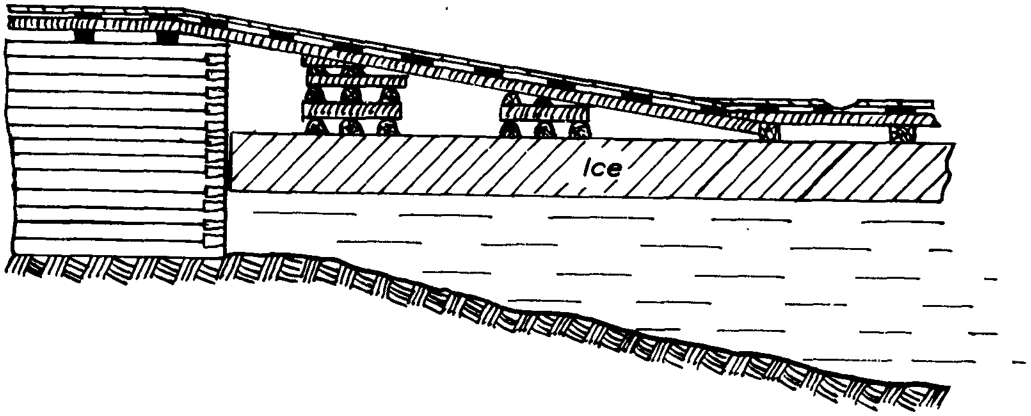

Fig. 33. Scheme for layout of crossing from pier to ice.

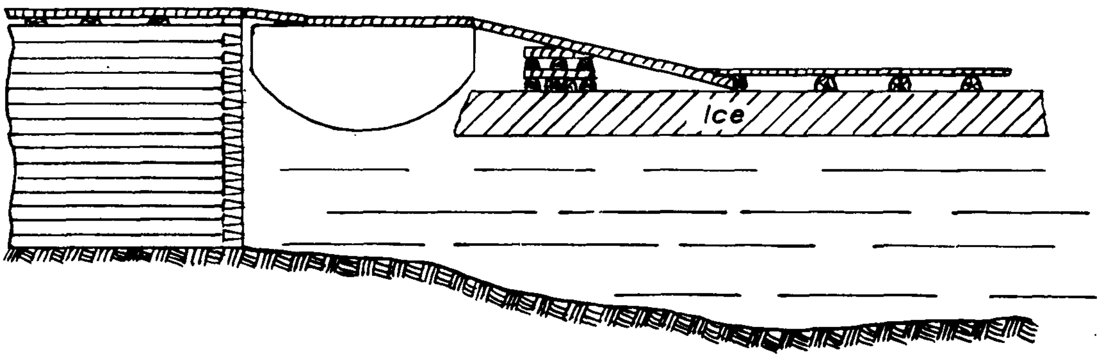

Fig. 34. Scheme for layout of crossing from pier to ice over a large barge.

The construction of railroad crossings is complicated by the fact that the spaces between rail joints should be small to avoid derailing. To meet the above requirements, local peculiarities of crossings, the availability of necessary materials, ground structure, bank elevation, etc., should be taken into consideration.

The following design for a railroad crossing over the Neva River has been projected. At the water's edge, close to the railroad embankment, a bank support of logs is planned. At some distance from the water's edge a double-columned wooden cribwork is to be built to receive a roadbed frame with one end fastened to the bank support and the other loose so as to be easily lowered down or raised by winches installed one on each column of the cribwork. The river end of the frame will connect with cross trusses to reinforce the ice cover. Winch-power calculations should provide for raising and lowering the frame when it is not loaded. Lifting cables are to be sufficiently strong to support the weight of the roadbed frame and the passing trains. On the inside edges of the cribwork columns special cable-rollers are to be provided to keep the roadbed frame level. A general construction diagram is shown in Fig. 35.

Calculations for the bank reinforcement may be found in mechanical engineering manuals. This problem is not dealt with in the present study.

6. Construction of crossings over fissures.

Removing snow from the ice crossing increases thermal tensions in the ice cover. They reach a maximum in the traffic lane. In conjunction with stresses from the traffic load and the lateral snow banks they cause fissures to form. Small, superficial fissures crossing the road in various directions appear at the beginning of operations. Further operations cause the formation of zigzag fissures along the crossing and ice shearing in some places. Fissures across the route sometimes reach a width of a few cm. to a few m. There is no regularity in the appearance of fissures or their form, and they are only detected in places where the ice has suddenly weakened. Very rarely do they appear in the same places.

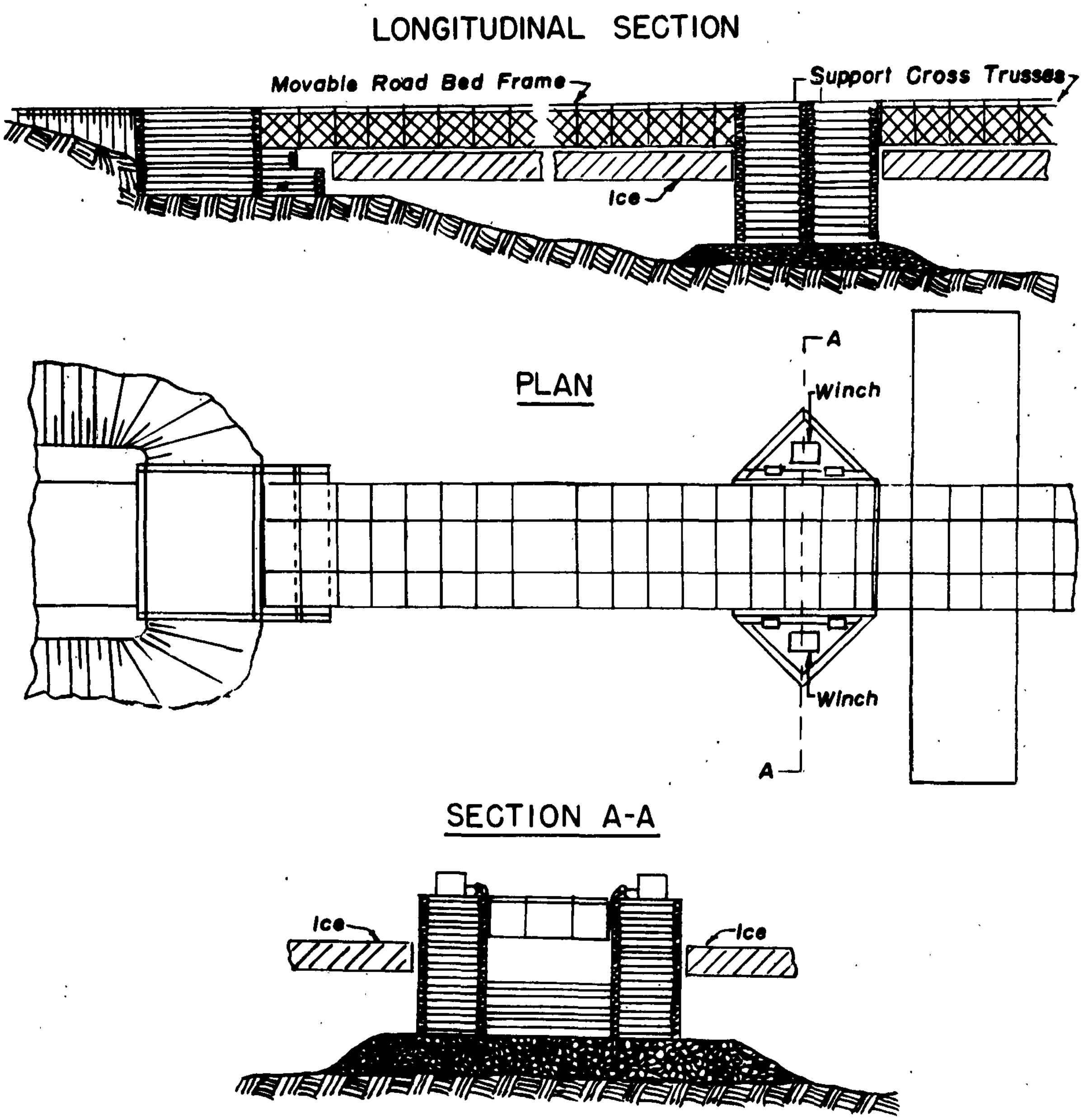

Fig. 35. Diagram of the construction from frames of a movable roadbed from the bank to the ice.

Bridges of two or more beams covered with a flooring are usually built to cross the fissures. Boards are used to cross small fissures. The latter are covered with snow, flooded and frozen.

The bridge should be considerably longer than the fissure is wide, since the ice at the main fissure's edge is weakened by the presence of many smaller fissures. The bridge must be calculated so as to be able to carry the maximum load planned for the whole crossing. Fissures that require bridging are the product of thermal tension in the ice cover and consequently change their width with changes in temperature. Therefore, one of the supports must be movable. Bridge supports should never be frozen into the ice. Non-observance of this rule may cause damage. For example, during the winter of 1941-42, a bridge for heavy tank traffic was constructed over a fissure in the ice cover of Lake Ladoga. Some time later another fissure was discovered, which circumvented the bridge from one side and joined the first. The bridge with its supports fastened to the ice prevented thermal ice compression, consequently the thermal tension was greater than the ultimate strength of the ice around the bridge and a block of ice broke off. Fortunately, there were no tanks crossing at that time.

The second condition to be remembered when building a bridge over a fissure is the load distribution along the fissure's edge. As shown before, the bearing capacity of the ice cover diminishes as the load approaches the fissure's edge, and is least when the load is close to the edge. The problem, consequently, is to find a load distribution under which the actual stress will not surpass the allowable stress. Therefore foundation beams should be lengthened. Since the bridge should not be fastened on both sides it is possible to make beams on one side slide on sleepers frozen into the ice. Such a bridge is shown in Fig. 36.

On one side of the fissure a snow bank is built on which cross beams are rested. The snow is flooded and the beams are frozen into the snow. On the other side of the fissure beams rest on transverse logs frozen into the ice. Logs and beams should not be fastened together rigidly. The flooring is laid across the beams. The area to be covered by logs as well as the necessary beam length are calculated according to equations (77), (86) and (94).

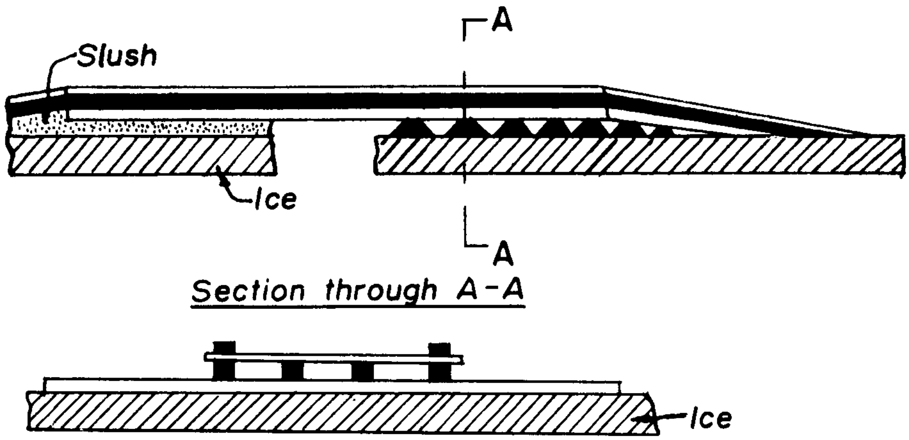

Fig. 36. Construction of a bridge over a thermal fissure.

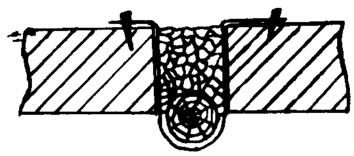

Fig. 37. Closing a 30-cm. wide fissure by means of a log placed under the surface of the ice.

The approaches to these bridges are reinforced by additional freezing; brushwood is covered with snow, flooded and frozen, or a bed is added under the tracks.

Instead of building a bridge, the fissure may be frozen. B. V. Proskuriakov suggested the use of ice to close fissures. His suggestions were experimentally tested during the winter of 1941-42 in an ice crossing, and gave good results. Proskuriakov's suggestions were as follows.

If the fissure winds within the limits of the traffic lane, it should be straightened out with pickaxes or saws. It should be done in sections, in a broken line, thus avoiding a widening of the fissure. If the width of the fissure is not more than 30 cm., it is possible to place a log under the ice and secure it from both ends to the fissure's edge by a rope or wire as is shown in Fig. 37. Poles or planks, fastened together, may replace the logs. After this is done, the fissure is packed with layers of crushed ice. The ice is flooded to accelerate the process of freezing.

Filling and ramming are started at one end of the fissure and continued step-by-step to the other end. Special attention should be paid to careful ramming since it determines the strength of the packing. This method can only be applied at air temperatures below -7°C and to a fissure not wider than 30-40 cm. Preparing the ice and filling a 3 m.-long fissure can be done by 4 men in 1 hour.

This work can best be achieved by a detachment of 5 men supplied with the following equipment:

1. Pickaxes -- 2
2. Axe -- 1
3. Cross-cut Saw -- 1
4. Crowbars -- 2
5. Wooden shovels -- 2
6. Buckets -- 2

Another method can be used to fill fissures up to 60 cm. wide. Edges of fissures are sheared off as shown in Fig. 38. Meanwhile, at some distance from the crossing wedge-shaped ice blocks are cut out, according to the width of the fissure, and put into the fissures.

These ice blocks fit the fissure closely and can carry loads even before they are frozen to the ice.

It is possible to close wide fissures (up to 3 m.) as well as bomb or shell craters with ice slabs. The edges of the fissure are straightened out with pickaxes or rip saws. At the same time, at some distance from the crossing, ice slabs, a little smaller than the fissure, are cut out. Three logs, up to 4 m. long, are put under the ice slab. A cable is fastened to the ends of the logs, and then the ice panel together with the logs is lowered down into the fissure. The ends of the cables are secured to logs 6 m. long, located under the ice as shown in cross section in Fig. 39 and in Fig. 40. Grooves between the fissure's edges and the ice panel are filled with crushed ice or snow and frozen.

The above method has never been tested experimentally. One can reckon, approximately, that a detachment of 10 men, equipped in a suitable manner, can close a 3-m. fissure in 5 hrs. The advantage of this method is that the work can be performed at relatively high air temperatures, from -3° to -5°C.

Ice panels may be cut out with pickaxes, rip saws or with the Red Army's powered handsaw MP-200.

After the fissure has been filled, the superstructure can be repaired, if necessary. The area over the filled fissure should be protected from direct load pressure by adding beams to the superstructure.

No less dangerous for traffic are sections of ice crisscrossed by small fissures. For safety, such areas are covered with a rod mat, tied together with a hemp rope.

The mat is covered with snow, flooded and frozen.

Two workers, preparing a 1-sq. m. mat, require:

hemp rope -- 6 m.

rods 3-5 cm. thick with a total length of 20 m.

The formation of fissures could probably be prevented by having thermal seams parallel to the traffic lane. For this purpose holes* are chopped into the ice at a distance of about 50 m. from the crossing, thus weakening the ice cover. Because of the thermal expansion of the ice, formation of fissures is to be expected near the holes and not on the road. Instead of chopping holes, the ice can be blasted with ammonal, ammonite, TNT, etc. Dynamite should not be used in freezing weather. The blasting procedure is: first, the charges are

* Footnote: Interval between holes about 10 m.

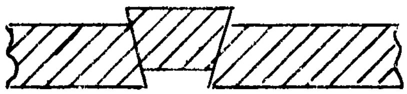

Fig. 38. A 60 cm.- fissure filled with ice blocks.

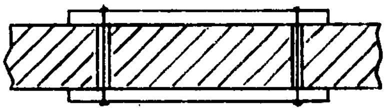

Fig. 39. A 3.0 m.-fissure filled with ice slabs, secured with logs.

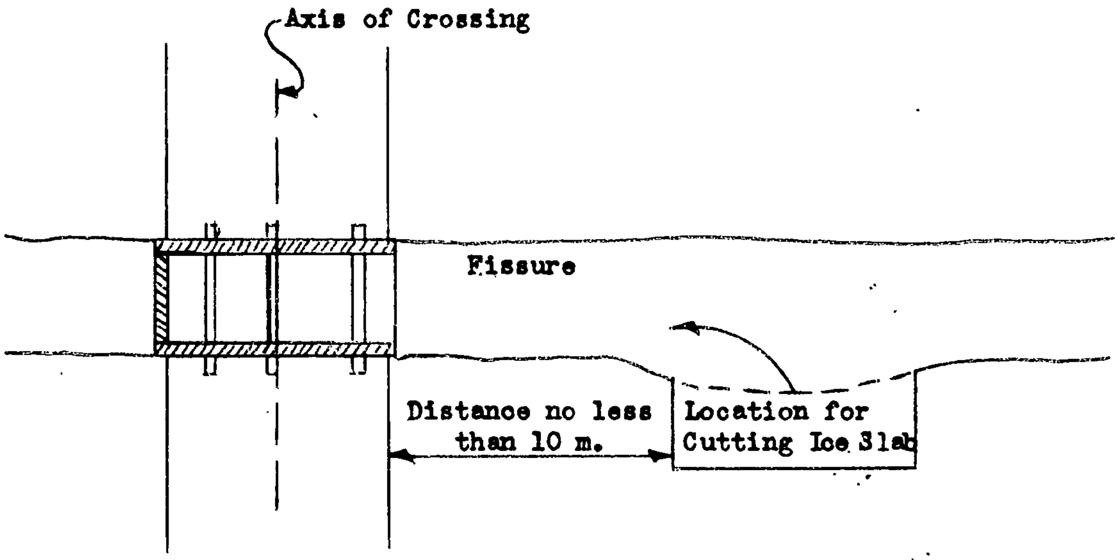

Fig. 40. Scheme of cutting out ice slabs for the closing of fissures.

distributed in places where the holes should be and connected with a fuse, then all charges are blasted synchronously. This method has never been used in practice. Nevertheless, thermal seams may be recommended as a protective measure against fissure formation.

7. Calculations of the supporting power of the ice cover and reinforcement of the crossing.

1. When loads are transported over a natural ice cover, the following cases should be differentiated:

1) load transport at small time intervals,

2) transport of separate loads.

2. In continuous operations, if the distance between the loads is more than 50 m., the calculation is made analogous to transportation of separate loads.

3. For traffic with loads one after another at small time intervals, the safe load is calculated from equation (141):

$$Q = \frac{\sigma r^2 \sqrt{h}}{0.87\sqrt{E} \; e^{-\beta r} \sin \beta r}$$

where: σ = allowable stress in tons/sq. m., determined in Diagram 3 (p.25) from the air temperature

r = radius of load distribution, determined from the relation

$r = 0.565\sqrt{F}$, where F is the area occupied by the load,

h = ice thickness in m.,

E = the modulus of elasticity in tons/sq. m. To be found according to the graph shown in Fig. 2.

e = the base of the natural system of logarithms, equal to 2.71,

$\beta = \sqrt[4]{\frac{1}{4EI}}$

I = the moment of inertia, equal to $\frac{h^3}{12}$

The bearing capacity of the ice cover for separate loads is determined from equation (71):

$$Q = \frac{2.57\sigma r^2 \sqrt{h}}{\sqrt{E} \; C_2 \left(\frac{r}{l}\right)}$$

The symbols are the same as in the preceding case. Only the function C_2 is added, which is determined from the relation between load distribution and ice characteristics. (See appendix). According to equation (71), the value,

$$l = \sqrt[4]{\frac{Eh^3}{12} \frac{m^2}{(m^2-1)}}$$

5. Safe distance between the load and the fissure's edge is determined from equations (122) and (125):

$$\sigma = \frac{PE\ h\ \beta\ e^{-\beta x}\ \sin\beta x\ \ C_2\left(\frac{r}{l}\right)}{2.24r^2\left[e^{-\beta a}\left(2EI\beta^3 - \frac{K}{2\beta}\right)(\cos\beta a - \sin\beta a) + \frac{K}{2\beta}\right]},$$

$$\alpha = \frac{0.787 + \frac{\pi}{2} n}{\beta},$$

where: a = distance of the load from the fissure's edge

n = any integer.

The method of selecting the closest values is explained in Chapter IV, Paragraphs 3 and 8.

6. When ice is not thick enough and a reinforcement is needed, the thickness and width of the additional layer is determinded from tables in Paragraph 2 or Chapter IV.

7. The speed of additional freezing is determined from the equation:

$h = \frac{\alpha\theta}{720}$ where: h = thickness of the ice layer in cm., frozen in 1 hr.,

α = coefficient of heat emission in kg. cal./sq. m./hr./°C,

θ = air temperature, as determined from the equation:

$$\alpha = 28 + \theta.$$

In the presence of wind, the coefficient of heat emission increases in the ratio $\sqrt{v} + 0.3$, where v = wind velocity in m./sec.

8. A superstructure is built when the required reinforcement cannot be reached by additional freezing. The required width and hardness of the structure are determined by calculation.

First the radius of load distribution is calculated from the equation:

$$r = \frac{Q\sqrt{E}\, C_2 \left(\frac{r}{l}\right)}{2.57\sigma\sqrt{n}}$$

The meaning of the symbols is explained above.

The area of load distribution is:

$$F = \pi r^2$$

At the expense of an increase in load distribution along the crossing, it is possible to diminish the width of the superstructure in conformity with:

$$a \times b = F$$

where: a = length of load distribution
b = width
F = area of distribution.

The hardness is determined from the maximum value of a or b. For this purpose, ice deflection under load at a distance of $\frac{a}{2}$ from the load center, with load distributed over area F, is found.

The equation of deflection is:

$$y = q\left[1 + C_1 Z_1 (x) + C_2 Z_2 (x)\right],$$

where: C_1 and C_2 are constants determined from the relation $\frac{Q}{\pi r^2}$, values of which are given in tables in the appendix; $Z_1 (x)$ and $Z_2 (x)$ are functions, determined from the relation between the distance of the point under study from the load center and the value l, from appendix 1.

q is the intensity of loading, equal to $\frac{Q}{\pi r^2}$

The difference in deflection o y_1 and y_2 is substituted in equation:

$$\Delta I = \frac{q x^2}{8 E J},$$

where: $x = \frac{a}{2}$

From here the value of the required moment of inertia is found and consequently the size of the superstructure, because $I = \frac{h^3}{12}$. The calculated height and materials available determine the type of superstructure to be constructed.

First the radius of load distribution is calculated from the equation:

$$r = \frac{Q\sqrt{E}\, C_2\, (\frac{r}{l})}{2.57\sigma\sqrt{n}}$$

The meaning of the symbols is explained above.

The area of load distribution is:

$$F = \pi r^2$$

At the expense of an increase in load distribution along the crossing, it is possible to diminish the width of the superstructure in conformity with:

$$a \times b = F$$

where: a = length of load distribution
b = width
F = area of distribution.

The hardness is determined from the maximum value of a or b. For this purpose, ice deflection under load at a distance of $\frac{a}{2}$ from the load center, with load distributed over area F, is found.

The equation of deflection is:

$$y = q\left[1 + C_1 Z_1(x) + C_2 Z_2(x)\right],$$

where: C_1 and C_2 are constants determined from the relation $\frac{Q}{\pi r^2}$, values of which are given in tables in the appendix; $Z_1(x)$ and $Z_2(x)$ are functions, determined from the relation between the distance of the point under study from the load center and the value l, from appendix 1.

q is the intensity of loading, equal to $\frac{Q}{\pi r^2}$

The difference in deflection of y_1 and y_2 is substituted in equation:

$$\Delta I = \frac{q x^2}{8EJ},$$

where: $x = \frac{a}{2}$

From here the value of the required moment of inertia is found and consequently the size of the superstructure, because $I = \frac{h^3}{12}$. The calculated height and materials available determine the type of superstructure to be constructed.

The values of the functions given in Appendices 1 thru 5 are used in the following equations:

(a) $W = q\left[1 + C_1 Z_1 (x) + C_2 Z_2 (x)\right]$

(b) $W = q\left[C_3 Z_3 (x) + C_4 Z_4 (x)\right]$

where W = the value of ice deflection under load in the loading zone in equation (a) and outside the loading zone in equation (b)

C_1, C_2, C_3 and C_4 are constants of integration, as functions of the ratio $\frac{r}{\ell}$, where r = radius of load distribution and ℓ = a numerical characteristic of the ice cover. $\ell = \sqrt[4]{\frac{m^2 Eh^3}{12m^2-1}}$

$Z_1 (x)$, $Z_2 (x)$, $Z_3 (x)$ and $Z_4 (x)$ are functions of the ratio $\frac{x}{\ell}$

$q = \frac{Q}{\pi r^2}$

Q = loading rate

APPENDIX 1

VALUES OF THE FUNCTIONS $z_1(\alpha)$, $z_2(\alpha)$, $z_3(\alpha)$, $z_4(\alpha)$

$\alpha = \frac{x}{l}$	$z_1(\alpha)$	$z_1'(\alpha)$	$z_2(\alpha)$	$z_2'(\alpha)$	$\alpha = \frac{x}{l}$	$z_3(\alpha)$	$z_3'(\alpha)$	$z_4(\alpha)$	$z_4'(\alpha)$
0	+1.0000	0	0	0	0	+0.5000	0	-	+
0.05	+1.0000	0	-0.0006	-0.0250	0.05	+0.4984	-0.0575	-1.9813	+12.7199
0.10	+1.0000	-0.0001	-0.0025	-0.0500	0.10	+0.4946	-0.0929	-1.5409	+ 6.3413
0.15	+1.0000	-0.0002	-0.0056	-0.0750	0.15	+0.4892	-0.1201	-1.2843	+ 4.2071
0.20	+1.0000	-0.0005	-0.0100	-0.1000	0.20	+0.4826	-0.1419	-1.1033	3.1340
0.25	+1.0000	-0.0010	-0.0156	-0.1250	0.25	+0.4751	-0.1598	-0.9640	2.4857
0.30	+0.9999	-0.0017	-0.0225	-0.1500	0.30	+0.4667	-0.1746	-0.8513	2.0498
0.35	+0.9998	-0.0027	-0.0306	-0.1750	0.35	+0.4577	-0.1769	-0.7571	1.7355
0.40	+0.9996	-0.0040	-0.0400	-0.2000	0.40	+0.4480	-0.1970	-0.6765	1.4974
0.45	+0.9994	-0.0057	-0.0506	-0.2249	0.45	+0.4380	-0.2054	-0.6065	1.3101
0.50	+0.9990	-0.0078	-0.0625	-0.2499	0.50	+0.4275	-0.2121	-0.5449	1.1585
0.55	+0.9986	-0.0104	-0.0756	-0.2749	0.55	+0.4168	-0.2175	-0.4902	1.0330
0.60	+0.9980	-0.0135	-0.0900	-0.2998	0.60	+0.4058	-0.2216	-0.4413	0.9273
0.65	+0.9972	-0.0172	-0.1056	-0.3247	0.65	+0.3946	-0.2247	-0.3972	0.8367
0.70	+0.9962	-0.0214	-0.1224	-0.3496	0.70	+0.3834	-0.2268	-0.3574	0.7582
0.75	+0.9951	-0.0264	-0.1405	-0.3744	0.75	+0.3720	-0.2281	-0.3212	0.6894
0.80	+0.9936	-0.0320	-0.1599	-0.3991	0.80	+0.3606	-0.2286	-0.2883	0.6286
0.85	+0.9918	-0.0384	-0.1805	-0.4238	0.85	+0.3491	-0.2224	-0.2583	0.5744
0.90	+0.9898	-0.0456	-0.2023	-0.4485	0.90	+0.3377	-0.2276	-0.2308	0.5258
0.95	+0.9873	-0.0535	-0.2253	-0.4730	0.95	+0.3264	-0.2262	-0.2056	0.4819
1.00	+0.9844	-0.0624	-0.2496	-0.4974	1.00	+0.3151	-0.2243	-0.1825	0.4422
1.10	+0.9771	-0.0831	-0.3017	-0.5458	1.10	+0.2929	-0.2193	-0.1419	0.3730
1.20	+0.9676	-0.1078	-0.3587	-0.5935	1.20	+0.2929	-0.2193	-0.1419	0.3730
1.30	+0.9554	-0.1370	-0.4204	-0.6403	1.30	+0.2504	-0.2054	-0.0786	+ 0.2656
1.40	+0.9401	-0.1709	-0.4867	-0.6860	1.40	+0.2302	-0.1971	-0.0542	+ 0.2235
1.50	+0.9211	-0.2100	-0.5576	-0.7302	1.50	+0.2110	-0.1882	-0.0337	+ 0.1873
1.60	+0.8979	-0.2546	-0.6327	-0.7727	1.60	+0.1926	-0.1788	-0.0166	+ 0.1560
1.70	+0.8760	-0.3048	-0.7120	-0.8131	1.70	+0.1752	-0.1692	-0.0020	0.1290
1.80	+0.8367	-0.3612	-0.7953	-0.8509	1.80	+0.1588	-0.1594	+0.0094	0.1056
1.90	+0.1433	-0.1496	+0.0189	+0.0854	1.90	+0.1433	-0.1496	+0.0189	0.0854
2.00	+0.7517	-0.4931	-0.9723	-0.9170	2.00	+0.1289	-0.1399	+0.0265	0.0679
2.50	+0.4000	-0.9436	-1.4572	-0.9983	2.50	+0.0709	-0.0950	+0.0437	0.0114
3.00	-0.2214	-1.5698	-1.9376	-0.8804	3.00	+0.0326	-0.0586	+0.0427	-0.0137
3.50	-1.1936	-2.3361	-2.2832	-0.4353	3.50	+0.0104	-0.0326	+0.0335	-0.0200
4.00	-2.5634	-3.1346	-2.2927	+0.4912	4.00	+0.0014	-0.0152	+0.0230	-0.0200

APPENDIX 2

VALUES OF THE FUNCTION $C_1\left(\frac{r}{l}\right)$

$\alpha = \frac{r}{l}$	0	1	2	3	4	5	6	7	8	9
0	-1.0000	-0.9998	-0.9997	-0.9992	-0.9988	-0.9982	-0.9979	-0.9975	-0.9968	-0.9963
0.10	-0.9958	-0.9950	-0.9940	-0.9930	-0.9920	-0.9908	-0.9898	-0.9885	-0.9870	-0.9857
0.20	-0.9842	-0.9825	-0.9808	-0.9892	-0.9775	-0.9755	-0.9740	-0.9720	-0.9700	-0.9682
0.30	-0.9660	-0.9640	-0.9618	-0.9595	-0.9565	-0.9538	-0.9518	-0.9495	-0.9465	-0.9440
0.40	-0.9410	-0.9378	-0.9345	-0.9315	-0.9280	-0.9250	-0.9220	-0.9190	-0.9160	-0.9128
0.50	-0.909	-0.905	-0.902	-0.898	-0.895	-0.891	-0.887	-0.883	-0.878	-0.875
0.60	-0.872	-0.868	-0.864	-0.860	-0.857	-0.850	-0.849	-0.845	-0.841	-0.837
0.70	-0.834	-0.8295	-0.8254	-0.8210	-0.8164	-0.8126	-0.8081	-0.8035	-0.7995	-0.7946
0.80	-0.7903	-0.7862	-0.7814	-0.7768	-0.7724	-0.7680	-0.7632	-0.7586	-0.7538	-0.7496
0.90	-0.7451	-0.7402	-0.7361	-0.7313	-0.7261	-0.7218	-0.7178	-0.7136	-0.7084	-0.7035
1.00	-0.6482	-0.6435	-0.6889	-0.6818	-0.6790	-0.6748	-0.6690	-0.6650	-0.6600	-0.6550
1.10	-0.6500	-0.6450	-0.6400	-0.6450	-0.6390	-0.6340	-0.6190	-0.6129	-0.6070	-0.6015
1.20	-0.5965	-0.5910	-0.5853	-0.5800	-0.5741	-0.5689	-0.5640	-0.5588	-0.5537	-0.5482
1.30	-0.5432	-0.5379	-0.5322	-0.5274	-0.5226	-0.5176	-0.5128	-0.5068	-0.5170	-0.4962
1.40	-0.4903	-0.4848	-0.4792	-0.4739	-0.4686	-0.4632	-0.4664	-0.4560		

APPENDIX 3

VALUES OF THE FUNCTION $C_2\left(\frac{r}{l}\right)$

$\alpha = \frac{r}{l}$	0	1	2	3	4	5	6	7	8	9
0	0	0.0066	0.0015	0.0025	0.0037	0.0051	0.0067	0.0085	0.0104	0.0124
0.10	0.0146	0.0171	0.0197	0.0224	0.0253	0.0282	0.0315	0.0349	0.0383	0.0416
0.20	0.0447	0.0481	0.0517	0.0553	0.0585	0.0619	0.0659	0.0698	0.0739	0.0779
0.30	0.0820	0.0858	0.0898	0.0938	0.0980	0.1028	0.1070	0.1114	0.1155	0.1193
0.40	0.1243	0.1281	0.1320	0.1350	0.1390	0.1450	0.1490	0.1530	0.1570	0.1610
0.50	0.165	0.169	0.174	0.178	0.182	0.185	0.190	0.194	0.198	0.202
0.60	0.207	0.211	0.217	0.221	0.225	0.230	0.234	0.238	0.241	0.245
0.70	0.249	0.259	0.256	0.260	0.264	0.268	0.272	0.276	0.279	0.282
0.80	0.285	0.289	0.293	0.296	0.300	0.304	0.307	0.311	0.315	0.318
0.90	0.322	0.322	0.325	0.328	0.332	0.335	0.339	0.342	0.346	0.349
1.00	0.355	0.358	0.362	0.366	0.369	0.372	0.376	0.379	0.382	0.386
1.10	0.388	0.391	0.395	0.398	0.400	0.403	0.406	0.409	0.411	0.414
1.20	0.416	0.418	0.420	0.422	0.424	0.426	0.428	0.429	0.430	0.431
1.30	0.432	0.434	0.435	0.436	0.437	0.437	0.438	0.439	0.440	0.440
1.40	0.441	0.441	0.441	0.442	0.442	0.442	0.442	0.442	0.442	0.442
1.50	0.442	0.442	0.442	0.442	0.442	0.442	0.442	0.442	0.442	0.442
1.60	0.442	0.442	0.442	0.442	0.442	0.442	0.442	0.442	0.442	0.442
1.70	0.442	0.442	0.442	0.442	0.442	0.442	0.442	0.442	0.442	0.442
1.80	0.442	0.442	0.442	0.442	0.442	0.442	0.442	0.442	0.442	0.442
1.90	0.442	0.442	0.442	0.442	0.442	0.442	0.441	0.441	0.441	0.441
2.00	0.441	0.441	0.440	0.440	0.440	0.440	0.439	0.439	0.438	0.438
2.10	0.437	0.436	0.435	0.434	0.433	0.433	0.432	0.431	0.430	0.429
2.20	0.427	0.426	0.425	0.424	0.422	0.421	0.419	0.417	0.416	0.415
2.30	0.413	0.412	0.410	0.408	0.407	0.405	0.403	0.407	0.399	0.397
2.40	0.395	0.393	0.391	0.389	0.387	0.385	0.383	0.381	0.379	0.377
2.50	0.375	0.373	0.371	0.369	0.367	0.366	0.364	0.362	0.360	0.358
2.60	0.356	0.354	0.356	0.350	0.348	0.347	0.345	0.343	0.340	0.338
2.70	0.336	0.334	0.332	0.330	0.328	0.326	0.324	0.321	0.319	0.317
2.80	0.315	0.314	0.312	0.310	0.308	0.306	0.304	0.302	0.300	0.298
2.90	0.296	0.294	0.292	0.290	0.287	0.285	0.283	0.281	0.279	0.277
3.00	0.275	0.273	0.270	0.268	0.266	0.264	0.261	0.259	0.256	0.254
3.10	0.252	0.250	0.247	0.245	0.243	0.241	0.239	0.236	0.234	0.231
3.20	0.229	0.226	0.224	0.221	0.219	0.217	0.214	0.212	0.209	0.207
3.30	0.205	0.202	0.200	0.197	0.195	0.192	0.190	0.187	0.185	0.182
3.40	0.179	0.177	0.174	0.174	0.168	0.165	0.163	0.161	0.159	0.157
3.50	0.155	0.152	0.149	0.146	0.143	0.141	0.139	0.136	0.133	0.130
3.60	0.127	0.124	0.121	0.118	0.115	0.113	0.110	0.108	0.106	0.103
3.70	0.100	0.097	0.094	0.091	0.088	0.085	0.082	0.079	0.076	0.073
3.80	0.070	0.066	0.063	0.059	0.055	0.051	0.048	0.044	0.041	0.037
3.90	0.033	0.030	0.027	0.024	0.021	0.018	0.015	0.011	0.008	0.005
4.00	0.002									

APPENDIX 4

VALUES OF THE FUNCTION $C_3\left(\frac{r}{l}\right)$

$\alpha = \frac{r}{l}$	0	1	2	3	4	5	6	7	8	9
0	0.0000	0.0010	0.0020	0.0025	0.0030	0.0039	0.0048	0.0054	0.0062	0.0070
0.10	0.0078	0.0090	0.0103	0.0128	0.0150	0.0180	0.0211	0.0238	0.0260	0.0290
0.20	0.0319	0.0346	0.0380	0.0420	0.0457	0.0500	0.0538	0.0577	0.0619	0.0668
0.30	0.0717	0.0761	0.0809	0.0853	0.0903	0.0953	0.1013	0.1071	0.1134	0.1197
0.40	0.1258	0.1313	0.1384	0.1470	0.1552	0.1640	0.1730	0.1813	0.1900	0.1993
0.50	0.2079	0.2170	0.2257	0.2339	0.2430	0.2523	0.2608	0.2693	0.2780	0.2872
0.60	0.2958	0.3047	0.3138	0.3227	0.3318	0.3400	0.3500	0.3598	0.3670	0.3754
0.70	0.3844	0.4000	0.4180	0.4310	0.4488	0.4647	0.4800	0.4985	0.5129	0.5288
0.80	0.5448	0.5618	0.5778	0.5940	0.6100	0.6264	0.6435	0.6598	0.6760	0.6820
0.90	0.7084	0.7258	0.7420	0.7583	0.7744	0.7910	0.8080	0.8240	0.8400	0.8580
1.00	0.8744	0.8910	0.9093	0.9258	0.9420	0.9594	0.9746	0.9918	1.0080	1.0249
1.10	1.0426	1.0598	1.0748	1.0910	1.1078	1.1238	1.1400	1.1559	1.1717	1.1880
1.20	1.2043	1.2203	1.2360	1.2530	1.2695	1.2860	1.3020	1.3183	1.3350	1.3598
1.30	1.3678	1.3840	1.4000	1.4173	1.4338	1.4500	1.4660	1.4824	1.4990	1.5160
1.40	1.5320	1.5488	1.5657	1.5820	1.5998	1.6160	1.6328	1.6500		

APPENDIX 5

VALUES OF THE FUNCTION $C_4\left(\frac{r}{l}\right)$

$\alpha = \frac{r}{l}$	0	1	2	3	4	5	6	7	8	9
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.20	0.0001	0.0002	0.0002	0.0003	0.0004	0.0005	0.0006	0.0007	0.0008	0.0008
0.30	0.0009	0.0010	0.0012	0.0012	0.0014	0.0015	0.0017	0.0019	0.0021	0.0023
0.40	0.0025	0.0028	0.0030	0.0032	0.0035	0.0038	0.0041	0.0044	0.0047	0.0051
0.50	0.0055	0.0060	0.0066	0.0072	0.0079	0.0080	0.0094	0.0102	0.0110	0.0118
0.60	0.0126	0.0136	0.0145	0.0154	0.0166	0.0176	0.0186	0.0196	0.0207	0.0219
0.70	0.0235	0.0280	0.0330	0.0380	0.0430	0.0486	0.0540	0.0590	0.0642	0.0698
0.80	0.0748	0.0800	0.0855	0.0908	0.0960	0.1012	0.1068	0.1120	0.1174	0.1228
0.90	0.1283	0.1348	0.1394	0.1450	0.1500	0.1554	0.1610	0.1664	0.1720	0.1770
1.00	0.1827	0.1880	0.1933	0.1888	0.2040	0.2092	0.2148	0.2200	0.2253	0.2309
1.10	0.2363	0.2419	0.2470	0.2527	0.2580	0.2638	0.2698	0.2750	0.2810	0.2869
1.20	0.2930	0.2989	0.3048	0.3107	0.3169	0.3225	0.3285	0.3344	0.3400	0.3460
1.30	0.3520	0.3580	0.3640	0.3697	0.3755	0.3814	0.3870	0.3980	0.3988	0.4046
1.40	0.4108	0.4163	0.4223	0.4280	0.4350	0.4413	0.4480	-	-	-

APPENDIX 6

Weight and Size of Loads

No.	Type of load	Distance between axles (Meters)	Axle length (Meters)	Weight in Metric Tons	
				Total	Maximum on an axle
1	Armed soldier	-	-	0.10-0.11	-
2	Civilian cart	1.2	1.2	0.6	0.3
3	Military cart	-	1.5	0.5	0.5
4	Ammunition cart	-	1.5	0.8	0.8
5	2-horse military cart	1.96	1.5	1.06	0.54
6	76 mm. cannon	3.97	1.5	2.0	1.065
7	122 mm. howitzer	4.27	1.55	2.4	1.475
8	107 mm. cannon	3.97	1.75	2.5	2.16
9	152 mm. howitzer	3.97	1.64	3.0	2.785
10	car	3.45	1.7	2.7	1.5
11	Truck (1.5 ton capacity)	3.07	1.8	3.5	2.75
12	Truck (3 ton capacity)	4.12	1.8	7.0	4.5
13	Truck (5 ton capacity)	4.8	1.85	10.0	7.0
14	Tractor ChTZ-60	4.0	2.5	10	-
15	Tractor CHTZ-65	4.08	2.4	11.2	-
16	Tractor ChTZ SG-65	-	-	12	-
17	Switch engine constructed by Kaluga factory 0-2-0	-	0.75	8	-
18	Switch engine " " " " "	-	0.75	14	-
19	Switch engine constructed by Odessa factory "	-	0.75	6	-
20	Switch engine " " " " "	-	0.75	16	-
21	Locomotive constructed by Kolomna factory 0-4-0	-	0.75	16	-
22	Locomotive " " " " "	-	0.75	26	-
23	Locomotive 0-4-0 Series OV	-	1.524	52.5	-
24	Freight car with 2 axles	-	-	-	-
25	Freight car with 4 axles	-	-	-	-
26	Pullman car	-	-	-	-
27	Tank car with 4 axles	-	-	-	-

APPENDIX 8

Tables for calculating the supporting power of an ice cover.

The supporting power of an ice cover according to equation (47) is expressed:

$$Q = \frac{2.56 \times \sigma \times r^2 \times h}{\sqrt{E} \times C_2\left(\frac{r}{l}\right)} = 2.56 \times \sigma \times S$$

where $S = \dfrac{\sqrt{h} \times r^2}{\sqrt{E} \times C_2\left(\frac{r}{l}\right)}$ is given in tables

Thus, for calculating the supporting power of an ice cover (Q) for a given value (E), radius of load distribution (r) and ice thickness (h), the value S is determined from the table and multiplied by $\dfrac{2.56 \times \sigma}{1000}$, where σ is the limit of safe bending stress of ice.

Modulus of ice elasticity E

Ice thickness (h) in m.	$0.5.10^5$	$1.0.10^5$	$2.0.10^5$	$3.0.10^5$	$4.0.10^5$	$5.0.10^5$	$6.0.10^5$	$7.0.10^5$	$8.0.10^5$	$9.0.10^5$	$10.0.10^5$
	Values of S for different radii of load distribution r = 0.5 m										
0	0	0	0	0	0	0	0	0	0	0	0
0.10	4.5	4.5	4.3	4.0	3.7	3.5	3.5	3.4	3.4	3.3	3.3
0.20	11.0	10.3	9.7	9.3	9.1	8.9	8.7	8.6	8.5	8.4	8.3
0.30	20.5	20.2	19.0	18.0	17.9	16.5	16.1	15.8	15.5	15.2	15.0
0.40	35.7	34.0	31.0	29.3	28.3	27.5	27.1	26.7	26.3	25.9	24.0
0.50	51.5	50.0	45.0	42.0	41.0	40.5	40.0	39.0	38.0	37.0	35.0
0.60	69.0	65.0	61.8	59.2	57.0	55.0	53.4	51.7	50.0	48.5	47.0
0.70	87.0	84.0	79.0	74.0	71.0	69.0	65.0	64.0	63.0	62.0	60.7
0.80	111.2	105.0	96.3	92.0	88.0	85.0	82.5	80.5	78.0	76.0	74.0
0.90	140.0	128.5	119.0	110.0	105.0	102.0	99.9	95.0	94.0	91.0	90.0
1.00	166.5	154.0	140.0	130.5	123.5	118.0	114.5	111.5	109.0	106.5	104.5
1.10	192.5	178.0	163.0	151.0	144.0	139.0	135.0	131.0	128.0	125.0	125.0
1.20	219.0	202.5	183.5	173.2	165.7	160.0	155.5	151.5	148.0	144.5	141.5
1.30	245.0	227.0	209.0	195.0	185.0	179.0	174.0	169.0	165.0	161.0	157.0
1.40	281.0	252.0	234.0	217.0	206.0	199.0	192.5	187.5	182.5	178.0	174.0
	Values of S for different radii of load distribution r = 1.0 m										
0	0	0	0	0	0	0	0	0	0	0	0
0.10	7.0	6.5	5.8	5.4	5.0	4.8	4.6	4.5	4.4	4.3	4.2
0.20	15.8	14.4	13.1	12.6	12.2	11.8	11.6	11.3	11.1	10.9	10.7
0.30	29.0	26.0	24.1	23.1	22.5	22.1	21.8	21.6	21.4	21.2	21.0
0.40	44.0	42.0	39.5	38.5	37.2	36.6	36.0	35.4	35.0	34.5	34.0
0.50	66.0	62.0	59.0	57.0	55.5	54.0	53.0	51.8	50.8	50.0	49.5
0.60	91.0	87.0	81.6	78.8	76.3	75.2	74.0	72.9	71.9	71.0	70.0
0.70	117.0	112.0	107.0	102.6	99.5	97.1	95.5	94.0	92.6	91.3	90.0
0.80	146.0	141.0	135.0	128.8	124.0	122.2	118.7	116.0	114.8	113.2	112.0
0.90	180.0	173.0	163.0	157.0	153.0	149.5	147.0	144.0	141.5	139.0	137.0
1.00	218.0	209.0	197.0	188.0	181.5	176.5	173.5	170.5	168.5	166.2	165.0
1.10	260.0	245.0	228.0	218.0	211.0	207.0	204.0	201.0	198.0	196.0	193.0
1.20	305.0	284.0	262.0	251.0	244.0	239.0	234.0	230.0	227.0	223.0	220.0
1.30	355.0	330.0	300.0	286.0	277.0	271.0	266.0	261.0	257.0	253.0	250.0
1.40	401.0	375.0	340.0	324.0	315.0	309.0	303.0	297.0	292.0	287.0	283.0
	Values of S for different radii of load distribution r = 2.0 m										
0	00	0	0	0	0	0	0	0	0	0	0
0.10	14.0	12.1	10.0	8.3	7.0	6.0	5.8	5.7	5.6	5.5	5.5
0.20	26.0	24.4	22.0	19.7	18.0	16.0	15.4	15.2	15.1	15.0	15.0
0.30	48.0	44.0	40.0	36.4	33.0	31.0	29.9	29.1	28.9	28.2	28.0
0.40	67.0	63.4	59.0	56.1	54.0	51.0	49.5	48.4	47.4	46.7	46.0
0.50	100.0	93.4	86.0	80.0	76.0	71.0	68.4	66.8	65.5	64.6	64.0
0.60	134.0	128.8	118.0	109.8	102.0	96.0	94.2	93.5	93.2	93.1	93.0
0.70	168.0	159.0	149.0	140.2	136.0	128.0	123.5	120.4	118.0	116.2	115.0
0.80	201.0	191.0	184.5	172.0	165.0	160.0	156.7	154.2	152.3	150.9	150.0
0.90	249.0	233.8	222.0	213.4	204.0	196.0	192.6	189.6	187.6	186.0	185.0
1.00	300.0	290.0	269.0	255.0	245.0	233.0	230.0	227.3	225.5	224.0	223.0
1.10	350.0	325.0	304.0	294.0	284.0	277.0	273.0	269.0	265.4	261.8	260.0
1.20	396.0	367.5	344.0	332.0	321.8	313.0	307.6	302.8	399.7	297.2	296.0
1.30	457.0	426.0	402.0	388.0	377.9	368.0	359.0	351.0	344.0	337.8	334.0
1.40	525.0	500.0	467.0	452.0	436.0	425.0	410.0	399.0	387.8	379.2	370.0

APPENDIX 8 (Continued)

Modulus of ice elasticity E

Ice thickness (h) in m.	$0.5.10^5$	$1.0.10^5$	$2.0.10^5$	$3.0.10^5$	$4.0.10^5$	$5.0.10^5$	$6.0.10^5$	$7.0.10^5$	$8.0.10^5$	$9.0.10^5$	$10.0.10^5$
	Values of S for different radii of load distribution r = 3.0 m										
0	0	0	0	0	0	0	0	0	0	0	0
0.10	20.0	17.0	14.0	13.0	11.8	11.0	10.2	9.5	9.3	9.1	9.0
0.20	42.8	35.5	29.5	26.5	24.4	23.6	22.7	22.0	21.5	21.0	20.7
0.30	71.0	59.2	50.0	46.0	43.2	41.0	40.0	39.0	37.7	37.0	36.0
0.40	99.7	80.3	74.6	69.2	66.3	63.3	61.8	60.5	59.5	58.5	57.3
0.50	134.5	114.5	103.0	96.5	93.0	91.0	88.7	87.0	85.5	84.0	83.0
0.60	173.5	156.5	136.5	128.5	122.5	119.0	116.5	114.5	112.5	110.0	110.0
0.70	210.0	187.5	172.0	165.0	160.0	155.0	152.0	149.0	147.0	145.0	143.0
0.80	253.0	231.7	217.3	207.0	200.0	194.0	189.3	186.0	182.7	180.5	178.0
0.90	310.0	285.0	261.0	249.0	241.0	235.0	230.0	225.0	221.5	218.0	216.0
1.00	371.0	344.0	312.0	297.0	287.0	278.0	273.0	268.0	265.0	261.5	258.0
1.10	431.0	386.0	362.0	350.0	340.0	330.0	322.0	315.0	311.0	308.0	305.0
1.20	491.0	453.0	417.0	399.0	388.0	379.0	373.0	368.0	364.0	359.0	354.0
1.30	563.0	519.0	478.0	461.0	445.0	433.0	425.0	419.0	414.0	410.0	407.0
1.40	644.0	594.0	544.0	514.0	500.0	490.0	481.0	474.0	468.0	463.0	457.0
	Values of S for different radii of load distribution r = 4.0 m										
0	0	0	0	0	0	0	0	0	0	0	0
0.10	33.0	25.5	20.4	18.5	17.0	15.9	14.9	14.1	13.3	12.5	12.0
0.20	68.0	59.0	43.0	36.8	33.0	31.0	29.8	28.8	28.1	27.4	27.0
0.30	100.0	77.6	66.0	59.0	55.0	53.4	52.0	50.8	49.8	48.8	48.0
0.40	132.0	117.0	95.0	88.0	82.0	79.0	76.8	75.0	73.0	71.5	70.0
0.50	174.0	172.0	125.0	119.0	115.0	110.0	108.3	105.8	113.5	111.4	100.0
0.60	224.0	194.0	160.5	143.0	138.3	137.0	136.0	135.2	134.6	134.2	134.0
0.70	266.0	241.0	202.0	189.8	182.3	178.0	174.5	171.3	168.8	167.0	166.0
0.80	310.0	284.5	257.0	242.5	232.5	226.5	221.0	216.3	212.0	208.0	204.5
0.90	365.0	329.1	310.0	292.2	279.2	270.0	265.0	261.0	258.0	254.8	252.0
1.00	451.0	402.0	372.0	348.0	334.0	324.0	317.5	312.0	307.0	303.0	300.0
1.10	505.0	450.0	420.0	406.0	380.0	373.0	368.0	364.0	360.0	356.8	354.0
1.20	585.0	530.0	478.0	454.0	440.0	430.0	424.0	418.8	414.0	410.5	408.0
1.30	660.0	580.0	554.0	508.0	516.0	489.0	496.0	478.5	474.5	470.0	467.0
1.40	756.0	700.0	662.0	598.0	575.0	562.0	551.0	543.0	537.0	531.0	528.0
	Values of S for different radii of load distribution r = 5.0 m										
0	0	0	0	0	0	0	0	0	0	0	0
0.10	48.0	36.0	30.0	24.0	21.5	19.0	18.5	18.0	18.0	18.0	18.0
0.20	113.0	79.5	61.0	47.2	42.3	39.7	37.5	36.0	35.0	34.1	33.4
0.30	134.0	106.5	90.0	76.0	69.0	63.0	60.0	60.0	58.5	58.0	56.0
0.40	170.5	133.0	115.0	104.0	97.8	92.3	89.0	86.5	84.5	83.0	81.1
0.50	222.5	180.0	155.0	143.0	132.0	126.0	121.5	117.0	116.0	115.0	114.0
0.60	272.0	230.0	195.0	184.0	171.0	169.0	164.0	160.5	157.5	154.5	152.5
0.70	316.0	275.0	237.0	226.0	214.0	210.0	206.0	202.0	199.0	196.0	194.0
0.80	368.0	339.0	297.0	279.0	265.0	253.0	247.5	242.5	238.5	235.0	232.0
0.90	420.0	394.5	347.0	330.0	312.0	308.0	296.0	292.0	290.0	284.0	280.0
1.00	535.0	466.0	421.0	390.0	373.0	360.0	353.0	346.0	340.0	334.2	329.0
1.10	582.0	534.0	484.0	448.0	428.0	418.0	412.0	395.0	394.0	390.0	383.0
1.20	685.0	617.0	549.0	517.0	498.0	483.0	473.5	466.5	460.0	455.0	450.0
1.30	770.0	698.0	622.0	588.0	572.0	558.0	540.0	526.0	520.0	518.0	514.0
1.40	874.0	780.0	707.0	665.0	650.0	620.0	607.0	597.0	587.0	578.0	570.0

APPENDIX 8 (Continued)

Ice thickness (h) in m.	Modulus of ice elasticity E										
	0.5.10^5	1.0.10^5	2.0.10^5	3.0.10^5	4.0.10^5	5.0.10^5	6.0.10^5	7.0.10^5	8.0.10^5	9.0.10^5	10.0.10^5
	Values of S for different radii of load distribution r = 6.0m										
0	0	0	0	0	0	0	0	0	0	0	0
0.10	-	-	-	-	-	-	-	-	-	-	20.0
0.20	-	-	90.0	72.0	61.0	53.0	50.0	48.0	44.0	43.0	42.0
0.30	200.0	160.0	114.0	92.0	82.0	80.0	75.0	73.0	71.0	70.0	68.0
0.40	234.0	172.0	141.0	127.0	118.0	110.0	105.0	102.0	100.0	99.0	97.0
0.50	280.0	224.0	184.0	166.0	154.0	148.0	144.0	142.0	140.0	138.0	136.0
0.60	334.0	270.0	230.0	212.0	203.0	195.0	190.0	188.0	185.0	183.0	181.0
0.70	392.0	323.0	284.0	264.0	250.0	236.0	230.0	225.0	220.0	215.0	212.0
0.80	440.0	384.0	340.0	306.0	300.0	283.0	275.0	269.0	266.0	263.0	260.0
0.90	510.0	459.0	404.0	373.0	354.0	340.0	330.0	323.0	317.0	312.0	308.0
1.00	600.0	524.0	474.0	442.0	418.0	400.0	389.0	382.0	376.0	373.0	370.0
1.10	688.0	612.0	544.0	509.0	484.0	464.0	450.0	443.0	437.0	430.0	428.0
1.20	792.0	674.0	624.0	584.0	556.0	534.0	522.0	513.0	507.0	502.0	496.0
1.30	886.0	785.0	720.0	668.0	632.0	606.0	595.0	588.0	585.0	584.5	584.0
1.40	10003.0	894.0	824.0	755.0	716.0	684.0	663.0	648.0	636.0	629.0	626.0
	Values of S for different radii of load distribution r = 7.0 m										
0	0	0	0	0	0	0	0	0	0	0	0
0.10	-	-	-	-	-	-	-	-	-	-	24.0
0.20	-	-	120.0	98.0	82.0	70.0	63.0	58.0	65.0	52.0	50.0
0.30	-	-	144.0	123.0	108.0	96.0	91.0	87.0	83.0	80.0	78.0
0.40	314.0	250.0	172.0	147.0	135.0	126.0	123.0	119.0	117.0	115.0	112.0
0.50	360.0	280.0	220.0	200.0	184.0	170.0	162.0	157.0	153.0	150.0	148.0
0.60	410.0	329.0	272.0	247.0	232.0	228.0	214.0	206.0	199.0	194.0	191.0
0.70	472.0	400.0	328.0	303.0	284.0	268.0	261.0	254.0	249.0	245.0	244.0
0.80	544.0	468.0	390.0	358.0	336.0	318.0	308.0	302.0	293.0	290.0	288.0
0.90	634.0	503.0	452.0	420.0	398.0	383.0	378.0	377.0	375.0	374.0	372.0
1.00	732.0	592.0	533.0	496.0	468.0	446.0	432.0	420.0	412.0	406.0	402.0
1.10	812.0	688.0	608.0	562.0	545.0	524.0	550.0	498.0	490.0	480.0	472.0
1.20	920.0	774.0	700.0	657.0	624.0	597.0	580.0	568.0	556.0	546.0	538.0
1.30	1016.0	880.0	792.0	726.0	693.0	670.0	652.0	640.0	628.0	616.0	604.0
1.40	1145.0	1010.0	926.0	854.0	794.0	750.0	727.0	710.0	696.0	683.0	676.0
	Values of S for different radii of load distribution r = 8.0 m										
0	0	0	0	0	0	0	0	0	0	0	0
0.10	-	-	-	-	-	-	-	-	-	-	-
0.20	-	271.0	152.5	118.5	102.0	90.7	84.5	79.5	74.5	69.0	65.0
0.30	-	284.0	176.0	114.0	128.0	120.0	116.0	112.0	104.0	97.0	96.0
0.40	403.0	290.0	205.0	175.0	159.0	149.0	146.0	143.0	140.0	132.0	125.0
0.50	440.0	320.0	236.0	220.0	200.0	192.0	190.0	188.0	182.0	170.0	162.0
0.60	456.0	380.0	311.0	281.0	262.0	249.0	240.0	232.0	223.0	216.0	210.0
0.70	502.0	455.0	380.0	336.0	305.0	300.0	295.0	289.0	275.0	266.0	251.0
0.80	612.0	531.0	440.0	404.0	375.0	359.0	356.0	340.0	328.0	320.0	314.0
0.90	708.0	600.0	512.0	470.0	444.0	424.0	414.0	396.0	380.0	376.0	366.0
1.00	833.0	693.0	592.0	543.0	518.0	492.0	475.0	461.0	451.0	443.0	435.0
1.10	916.0	780.0	675.0	616.0	594.0	570.0	544.0	528.0	508.0	505.0	500.0
1.20	1085.0	880.0	771.0	707.0	673.0	648.0	608.0	592.0	579.0	577.0	575.0
1.30	1135.0	980.0	860.0	792.0	752.0	726.0	698.0	676.0	660.0	658.0	654.0
1.40	1270.0	1089.0	961.0	885.0	853.0	812.0	786.0	765.0	748.0	736.0	725.0

APPENDIX 8 (Continued)

Modulus of ice elasticity E

Ice thickness (h) in m.	0.5.10^5	1.0.10^5	2.0.10^5	3.0.10^5	4.0.10^5	5.0.10^5	6.0.10^5	7.0.10^5	8.0.10^5	9.0.10^5	10.0.10^5
	Values of S for different radii of load distribution r = 9.0 m										
0	0	0	0	0	0	0	0	0	0	0	0
0.10	-	-	-	-	-	-	-	-	-	-	-
0.20	-	-	-	-	-	-	-	-	-	-	82.0
0.30	-	-	-	-	-	-	-	-	-	-	108.0
0.40	546.0	344.0	256.0	220.0	196.0	178.0	169.0	160.0	154.0	148.0	142.0
0.50	590.0	410.0	308.0	268.0	244.0	224.0	212.0	204.0	198.0	192.0	182.0
0.60	633.0	452.0	360.0	328.0	299.0	276.0	262.0	252.0	243.0	227.0	234.0
0.70	696.0	532.0	432.0	388.0	360.0	336.0	321.0	310.0	302.0	294.0	288.0
0.80	754.0	600.0	502.0	452.0	419.0	394.0	378.0	368.0	357.0	349.0	343.0
0.90	832.0	665.0	582.0	527.0	494.0	464.0	444.0	428.0	417.0	408.0	402.0
1.00	95[illegible].0	750.0	660.0	604.0	556.0	540.0	522.0	506.0	493.0	482.0	474.0
1.10	1048.0	845.0	754.0	690.0	649.0	618.0	596.0	580.0	566.0	553.0	540.0
1.20	1164.0	[illegible]	847.0	779.0	736.0	704.0	682.0	664.0	647.0	634.0	623.0
1.30	1276.0	1074.0	954.0	866.0	819.0	792.0	765.0	745.0	728.0	712.0	696.0
1.40	1434.0	1244.0	1063.0	970.0	913.0	880.0	850.0	826.0	808.0	792.0	778.0
	Values of S for different radii of load distribution r = 10.0 m										
0	0	0	0	0	0	0	0	0	0	0	0
0.10	-	-	-	-	-	-	-	-	-	-	-
0.20	-	-	-	226.0	179.0	150.0	136.0	125.0	115.0	107.0	101.0
0.30	-	-	-	240.0	200.0	176.0	170.0	160.0	150.0	138.0	130.0
0.40	715.0	462.0	319.0	261.0	226.0	205.0	212.0	200.0	190.0	175.0	158.0
0.50	746.0	506.0	360.0	303.0	270.0	253.0	256.0	242.0	235.0	217.0	214.0
0.60	783.0	555.0	403.0	355.0	325.0	310.0	298.0	287.0	277.5	268.0	261.0
0.70	837.0	608.0	472.0	424.0	392.0	373.0	364.0	342.0	336.0	322.0	316.0
0.80	905.0	678.0	559.0	506.0	465.0	443.0	426.0	408.0	396.0	386.0	375.0
0.90	975.0	764.0	644.0	584.0	540.0	510.0	500.0	480.0	462.0	453.0	442.0
1.00	1065.0	877.0	738.0	663.0	621.0	586.0	564.0	549.0	537.0	528.0	520.0
1.10	1166.0	970.0	824.0	748.0	702.0	675.0	648.0	626.0	612.0	604.0	588.0
1.20	1295.0	1095.0	926.0	837.0	788.0	770.0	728.0	708.0	688.0	681.0	668.0
1.30	1435.0	1200.0	1024.0	936.0	884.0	852.0	812.0	792.0	772.0	762.0	748.0
1.40	1600.0	1355.0	1130.0	1055.0	1005.0	949.0	916.0	888.5	867.0	849.0	837.0

Footnotes

1. $Q = \dfrac{2.56 \cdot \sigma \cdot r^2 \sqrt{h}}{\sqrt{E} \cdot C_2\left(\frac{r}{l}\right)}$; $S = \dfrac{Q}{2.56 \cdot \sigma} = \dfrac{\sqrt{h} \cdot r^2}{\sqrt{E} \cdot C_2\left(\frac{r}{l}\right)}$

2. Values in table were multiplied by 1000

3. All values are expressed in m,